LECTURE NOTES

ON

ELECTRICAL MACHINE-II

Subject Code - BEE 1401
For B-Tech 4th SEM EE & EEE
[Part-I]
[Module-I & II]

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Detailed Syllabus

Department of Electrical Engineering,
Veer Surendra Sai University of Technology, Orissa, Burla, India
Syllabus of Bachelor of Technology in Electrical Engineering, 2010

ELECTRICAL MACHINES-II (3-1-0)
(4th SEMESTER)
Subject Code: BEE 1401

MODULE-I (10 HOURS)
Fundamental Principles of A.C. Machines: E.M.F. equation of an elementary alternator, Single & three Phase, relation between speed & frequency, factors affecting the induced e.m.f., full pitch & fractional pitch windings, winding factors, armature reaction, the rotating field leakage reactance. Concept of time phasor & space phasor. Synchronous Generator: Various types & construction, cylindrical rotor theory, phasor diagram, open circuit & short circuit characteristics, armature reaction reactance, synchronous reactance, SCR, load characteristics, potier reactance, voltage regulation, EMF method, MMF method, modified MMF method, ZPF method, power angle characteristics.

MODULE-II (10 HOURS)

MODULE-III (10 HOURS)
Three Phase Induction Motors: Types, Construction and principle of operation, 3 phase Induction Motor, general phasor diagram, equivalent circuit, power and torque relations, condition for maximum torque, circle diagram, Performance characteristics, effect of rotor resistance on speed torque characteristics, stable & unstable region of operation, Operation with unbalanced supply voltage. Starting: Starting of 3 phase induction motors, high starting torque motors, speed control, rheostatic method, pole changing method cascade control of speed, Double cage induction motor, Cogging and Crawling of Induction motor, induction generator

MODULE-IV (10 HOURS)

BOOKS
Syllabus of Bachelor of Technology in Electrical Engineering  
*(4th SEMESTER)*

**ELECTRICAL MACHINES-II**
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| **MODULE-II** | |
SYLLABUS/ TOPICS COVERED

Fundamental Principles of A.C. Machines: E.M.F. equation of an elementary alternator, Single & three Phase, relation between speed & frequency, factors affecting the induced e.m.f., full pitch & fractional pitch windings, winding factors, armature reaction, the rotating field leakage reactance. Concept of time phasor & space phasor.

Synchronous Generator: Various types & construction, cylindrical rotor theory, phasor diagram, open circuit & short circuit characteristics, armature reaction reactance, synchronous reactance, SCR, load characteristics, potier reactance, voltage regulation, EMF method, MMF method, modified MMF method, ZPF method, power angle characteristics.

[Topics are arranged as per above sequence]
1.1 Fundamental Principles of A.C. Machines:

AC rotating machines can be classified mainly in two categories **Synchronous Machines** and **Asynchronous Machines**. They are defined as-

* **Synchronous Machines**:
  - Synchronous Generators: A primary source of electrical energy.
  - Synchronous Motors: Used as motors as well as power factor compensators (synchronous condensers).

* **Asynchronous (Induction) Machines**:
  - Induction Motors: Most widely used electrical motors in both domestic and industrial applications.
  - Induction Generators: This generator runs at asynchronous speed and variable frequency voltage generated. Due to lack of a separate field excitation, these machines are rarely used as generators.

1.2 E.M.F. equation of an elementary alternator single phase

Let us assume that this generator has an armature winding consisting of a total number of full pitched concentrated coils $C$, each coil having a given number of turns $N_c$. Then the total number of turns in any given phase of a single-phase generator armature is

$$N_p = CN_c$$

According to Faraday’s law of electromagnetic induction the average voltage induced in a single turn of two coil sides is

$$E_{av} = \frac{\phi}{t}$$

The voltage induced in one conductor is $2\phi/(1/n) = 2\phi s$, where $n=$speed of rotation in r.p.s, for a 2 pole generator. Furthermore, when a coil consisting of $N_c$ turns rotates in a uniform magnetic field, at a uniform speed, the average voltage induced in an armature coil is

$$\frac{E_{av}}{coil} = 4\phi N_c n \text{ volts}$$

where $\phi$ is the number of lines of flux (in Webers) per pole, $N_c$ is number of turns per coil, $n$ is the relative speed in revolutions/second (rps) between the coil of $N_c$ turns and the magnetic field $\phi$. 
A speed \( n \) of 1 rps will produce a frequency \( f \) of 1 Hz. Since \( f \) is directly proportional and equivalent to \( n \), (for a 2-pole generator) for all the series turns in any phase, 

\[
E_{av \ phase} = 4\phi N_p f \text{ volts}
\]

The effective rms value of a sinusoidal ac voltage is 1.11 times the average value. The effective ac voltage per phase is

\[
E_{eff} = 4.44\phi N_p f \text{ volts}
\]

1.3 E.M.F. equation of an elementary alternator three phase

Let us assume that this generator has an armature winding consisting of a total number of full pitched concentrated coils \( C \), each coil having a given number of turns \( N_c \). Then the total number of turns in any given phase of a 3-phase generator armature is

\[
N_p = \frac{CN_c}{3}
\]

Voltage equation per phase will be similar in to the single phase alternator

\[
E_{ph} = 4.44\phi N_p f
\]

The value of line voltage will be different from phase voltage in case of star connected generator. The line value of the emf in case of three phase alternator connected in star will be-

\[
E_L = \sqrt{3}E_{ph}
\]

The value of line voltage will be same with phase voltage in case of delta connected generator. The line value of the emf in case of three phase alternator connected in delta will be-

\[
E_L = E_{ph}
\]

1.4 Relation between speed and frequency

One complete revolution will produce one complete positive and negative pulse each cycle when the number of pole is two. The frequency in cycles per second (Hz) will depend directly on the speed or number of revolutions per second (rpm/60) of the rotating field.
If the ac synchronous generator has multiple poles (having, say, two, four, six, or eight poles...), then for a speed of one revolution per second (1 rpm/60), the frequency per revolution will be one, two, three, or four ..., cycles per revolution, respectively. The frequency per revolution, is therefore, equal to the number of pairs of poles. Since the frequency depends directly on the speed (rpm/60) and also on the number of pairs of poles (P/2), then these two may be combined together into a single equation in which

\[ f = \frac{P \times \text{rpm}}{2 \times 60} = \frac{PN}{120} \]

\[ \omega_m = \frac{2 \times \pi \times N}{60} \]

\[ N = \frac{\omega_m \times 60}{2\pi} \]

\[ f = \frac{P \times \omega_m}{2 \times 2\pi} = \frac{\omega_e}{2\pi} \]

Where

P is the number of poles

N is the speed in rpm (rev/min)

f is the frequency in hertz

\( \omega_m \) is the speed in radians per second (rad/s)

\( \omega_e \) is the speed electrical radians per second.

1.5 Factors affecting the induced emf (Coil Pitch and Distributed Windings)

The emf equation derived in art 1.2 and art 1.3 is applicable when the alternator is having full pitch coil and concentrated winding. But when the alternator armature winding is distributed and short pitched then the per phase emf equation will change and become-
\[ E_g = 4.44\phi N_p f k_p k_d \]

Where \( K_p \) is called pitch factor and \( K_d \) is called distribution factor.

### 1.5.1 Pitch Factor or Coil Pitch

The ratio of phasor (vector) sum of induced emfs per coil to the arithmetic sum of induced emfs per coil is known as pitch factor (\( K_p \)) or coil span factor (\( K_c \)) which is always less than unity.

Let the coil have a pitch short by angle \( \theta \) electrical space degrees from full pitch and induced emf in each coil side be \( E \).

**Fig: 1(a) Voltage phasor for short-pitch coil**

- If the coil would have been full pitched, then total induced emf in the coil would have been \( 2E \).
- when the coil is short pitched by \( \theta \) electrical space degrees the resultant induced emf, \( E_R \) in the coil is phasor sum of two voltages, \( \theta \) apart

\[
E_R = 2E \cos \frac{\theta}{2}
\]

Pitch Factor, \( K_p = \frac{\text{Phasor sum of coil side emfs}}{\text{Arithmetic sum of coil side emfs}} = \frac{2E \cos \frac{\theta}{2}}{2E} = \cos \frac{\theta}{2}
\]

The pitch factor of the coil at the \( n^{th} \) harmonic frequency can be expressed as

\[
k_{pm} = \cos \frac{n\theta}{2} \text{ where } n \text{ is the order of harmonic}
\]
1.5.2 Distribution Factor

The ratio of the phasor sum of the emfs induced in all the coils distributed in a number of slots under one pole to the arithmetic sum of the emfs induced (or to the resultant of emfs induced in all coils concentrated in one slot under one pole) is known as *breadth factor* \((K_b)\) or *distribution factor* \((K_d)\).

The distribution factor is always less than unity.

Let no. of slots per pole = \(Q\) and no. of slots per pole per phase = \(q\)

Induced emf in each coil side = \(E_c\)

Angular displacement between the slots, \(\gamma^0\).

The emf induced in different coils of one phase under one pole are represented by side AC, CD, DE, EF… Which are equal in magnitude (say each equal \(E_c\)) and differ in phase (say by \(\gamma^0\)) from each other.

If bisectors are drawn on AC, CD, DE, EF… they would meet at common point \((O)\). The point \(O\) would be the center of the circle having AC, CD, DE, EF…as the chords and representing the emfs induced in the coils in different slots.
EMF induced in each coil side, \( E_c = AC = 2OA \sin \frac{\gamma}{2} \)

Arithmetic sum = \( q \times 2 \times OA \sin \frac{\gamma}{2} \)

The resultant emf, \( E_R = AB = 2 \times OA \sin \frac{AOB}{2} \) & distribution factor,

The distribution factor for \( n \)th order harmonic component is given as

\[
k_{dn} = \frac{\sin \frac{nq\gamma}{2}}{q \sin \frac{ny}{2}}, \text{ where } n \text{ is the order of harmonic}
\]

1.5.3 Harmonic Effect

- The flux distribution along the air gaps of alternators usually is non-sinusoidal so that the emf in the individual armature conductor likewise is non-sinusoidal.
- The sources of harmonics in the output voltage waveform are the non-sinusoidal waveform of the field flux.
- Fourier showed that any periodic wave may be expressed as the sum of a d-c component (zero frequency) and sine (or cosine) waves having fundamental and multiple or higher frequencies, the higher frequencies being called harmonics.
- All the odd harmonics (third, fifth, seventh, night, etc.) are present in the phase voltage to some extent and need to be dealt with in the design of ac machines.
- Because the resulting voltage waveform is symmetric about the center of the rotor flux, no even harmonics are present in the phase voltage.
- In Y- connected, the third-harmonic voltage between any two terminals will be zero. This result applies not only to third-harmonic components but also to any multiple of a third-harmonic component (such as the ninth harmonic). Such special harmonic frequencies are called triplen...
**harmonics**

**Elimination or Suppression of Harmonics**

Field flux waveform can be made as much sinusoidal as possible by the following methods:

1. Small air gap at the pole centre and large air gap towards the pole ends
2. **Skewing**: skew the pole faces if possible
3. **Distribution**: distribution of the armature winding along the air-gap periphery
4. **Chording**: with coil-span less than pole pitch
5. Fractional slot winding
6. **Alternator connections**: star or delta connections of alternators suppress triplen harmonics from appearing across the lines

**1.5.4 Winding Factor**

Both distribution factor \(K_d\) and pitch factor \(K_p\) together is known as **winding factor** \(K_w\).

\[
k_w = k_p k_d
\]

\[
E_g = 4.44\phi N_p f \ k_w
\]

**1.6 Armature Reaction**

When an alternator is running at no-load, there will be no current flowing through the armature winding. The flux produced in the air-gap will be only due to the rotor ampere turns. When the alternator is loaded, the three-phase currents will produce a totaling magnetic field in the air-gap. Consequently, the air-gap flux is changed from the no-load condition.

The effect of armature flux on the flux produced by field ampere turns (i.e., rotor ampere turns) is called armature reaction.

Two things are worth noting about the armature reaction in an alternator. First, the armature flux and
the flux produced by rotor ampere-turns rotate at the same speed (synchronous speed) in the same direction and, therefore, the two fluxes are fixed in space relative to each other.

Secondly, the modification of flux in the air-gap due to armature flux depends on the magnitude of stator current and on the power factor of the load. It is the load power factor which determines whether the armature flux distorts, opposes or helps the flux produced by rotor ampere-turns.

To illustrate this important point, we shall consider the following three cases:

1. When load p.f. is unity
2. When load p.f. is zero lagging
3. When load p.f. is zero leading

*When load p.f. is unity*

![Fig: 1 (c)](image)

Above Fig: 1 (c) shows an elementary alternator on no load. Since the armature is on open-circuit, there is no stator current and the flux due to rotor current is distributed symmetrically in the air-gap as shown in Fig: 1 (d). Since the direction of the rotor is assumed clockwise, the generated e.m.f. in phase R1R2 is at its maximum and is towards the paper in the conductor R1 and outwards in conductor R2. No armature flux is produced since no current flows in the armature winding.

Fig (ii) shows the effect when a resistive load (unity p.f.) is connected across the terminals of the alternator. According to right-hand rule, the current is “in” in the conductors under N-pole and “out” in the conductors under S-pole. Therefore, the armature flux is clockwise due to currents in the top conductors and anti-clockwise due to currents in the bottom conductors. Note that armature flux is at
90° to the main flux (due to rotor current) and is behind the main flux.

In this case, the flux in the air-gap is distorted but not weakened. Therefore, at unity p.f., the effect of armature reaction is merely to distort the main field; there is no weakening of the main field and the average flux practically remains the same. Since the magnetic flux due to stator currents (i.e., armature flux) rotate; synchronously with the rotor, the flux distortion remains the same for all positions of the rotor.

**When load Power Factor is Zero lagging**

When a pure inductive load (zero p.f. lagging) is connected across the terminals of the alternator, current Fig: 1 (c) shows the condition when the alternator is supplying resistive load. Note that e.m.f. as well as current in phase R1R2 is maximum in the position shown. When the alternator is supplying a pure inductive load, the current in phase R1R2 will not reach its maximum value until N-pole advanced 90° electrical as shown in Fig: 1 (d). Now the armature flux is from right to left and field flux is from left to right All the flux produced by armature current (i.e., armature flux) opposes be field flux and, therefore, weakens it. In other words, armature reaction is directly demagnetizing. Hence at zero p.f. lagging, the armature reaction weakens the main flux. This causes a reduction in the generated e.m.f.

**When load Power Factor is Zero leading**

When a pure capacitive load (zero p.f. leading) is connected across the terminals of the alternator, the current in armature windings will lead the induced e.m.f. by 90°.

![Diagram](image.png)

Fig: 1 (d)

Obviously, the effect of armature reaction will be the reverse that for pure inductive load. Thus armature
flux now aids the main flux and the generated e.m.f. is increased. Fig: 1 (c) shows the condition when alternator is supplying resistive load.

Note that e.m.f. as well as current in phase R1R2 is maximum in the position shown. When the alternator is supplying a pure capacitive load, the maximum current in R1R2 will occur 90° electrical before the occurrence of maximum induced e.m.f. Therefore, maximum current in phase R1R2 will occur if the position of the rotor remains 90° behind as compared to its position under resistive load. This is illustrated in Fig: 1 (d). It is clear that armature flux is now in the same direction as the field flux and, therefore, strengthens it. This causes an increase in the generated voltage. Hence at zero p.f. leading, the armature reaction strengthens the main flux.

For intermediate values of p.f, the effect of armature reaction is partly distorting and partly weakening for inductive loads. For capacitive loads, the effect of armature reaction is partly distorting and partly strengthening. Note that in practice, loads are generally inductive.

1.7 Synchronous Generators

_Synchronous machines_ are principally used as _alternating current (AC) generators._

- They supply the electric power used by all sectors of modern societies: industrial, commercial, agricultural, and domestic. They

- usually operate together (or in parallel), forming a large power system supplying electrical energy to the loads or consumers.

- are built in large units, their rating ranging from tens to hundreds of megawatts.

- converts mechanical power to ac electric power. The source of mechanical power, _the prime mover_, may be a diesel engine, a steam turbine, a water turbine, or any similar device.

For high-speed machines, the prime movers are usually _steam turbines_ employing fossil or nuclear energy resources.

Low-speed machines are often driven by _hydro-turbines_ that employ water power for generation.
Smaller synchronous machines are sometimes used for private generation and as standby units, with diesel engines or gas turbines as prime movers.

1.7.1 Various Types of Synchronous Machine & Construction
According to the arrangement of the field and armature windings, synchronous machines may be classified as rotating-armature type or rotating-field type.

1.7.2 Rotating-Armature Type:
The armature winding is on the rotor and the field system is on the stator.

1.7.3 Rotating-Field Type:
The armature winding is on the stator and the field system is on the rotor.

According to the shape of the field, synchronous machines may be classified as cylindrical-rotor (non-salient pole) machines and salient-pole machines
AC winding design
The windings used in rotating electrical machines can be classified as

Concentrated Windings
- All the winding turns are wound together in series to form one multi-turn coil
- All the turns have the same magnetic axis
- Examples of concentrated winding are
  - field windings for salient-pole synchronous machines
  - D.C. machines
  - Primary and secondary windings of a transformer

Distributed Windings
- All the winding turns are arranged in several full-pitch or fractional-pitch coils
- These coils are then housed in the slots spread around the air-gap periphery to form phase or commutator winding
- Examples of distributed winding are
  - Stator and rotor of induction machines
  - The armatures of both synchronous and D.C. machines
Some of the terms common to armature windings are described below:

**Conductor.** A length of wire which takes active part in the energy-conversion process is called a conductor.

**Turn.** One turn consists of two conductors.

**Coil.** One coil may consist of any number of turns.

**Coil-side.** One coil with any number of turns has two coil-sides.

The number of conductors (C) in any coil-side is equal to the number of turns (N) in that coil.

![Fig: 1.1](image)

**Pole – pitch:** A pole pitch is defined as the peripheral distance between identical points on two adjacent poles. Pole pitch is always equal to $180^\circ$ electrical.

**Coil-span or coil-pitch:** The distance between the two coil-sides of a coil is called coil-span or coil-pitch. It is usually measured in terms of teeth, slots or electrical degrees.

**Chorded-coil**

- If the coil-span (or coil-pitch) is equal to the pole-pitch, then the coil is termed a **full-pitch coil**.

- in case the coil-pitch is less than pole-pitch, then it is called **chorded, short-pitch or fractional-pitch coil**
Fractional-pitch coil

In AC armature windings, the separate coils may be connected in several different manners, but the two most common methods are lap and wave.

1.7.2 Cylindrical Rotor Theory

Similar to the case of DC generator, the behavior of a Synchronous generator connected to an external load is different than that at no-load. In order to understand the performance of the Synchronous generator when it is loaded, consider the flux distributions in the machine when the armature also carries a current. Unlike in the DC machine in alternators the emf peak and the current peak will not occur in the same coil due to the effect of the power factor of the load. The current and the induced emf will be at their peaks in the same coil only for upf loads. For zero power factor lagging loads, the current reaches its peak in a coil which falls behind that coil wherein the induced emf is at its peak by 90 electrical degrees or half a pole-pitch. Likewise for zero power factor leading loads, the current reaches its peak in a coil which is ahead of that coil wherein the induced emf is at its peak by 90 electrical degrees or half a pole-pitch. For simplicity, assume the resistance and leakage reactance of the stator windings to be negligible. Also assume the magnetic circuit to be linear i.e. the flux in the magnetic circuit is deemed to be proportional.
to the resultant ampere-turns - in other words the machine is operating in the linear portion of the magnetization characteristics. Thus the emf induced is the same as the terminal voltage, and the phase-angle between current and emf is determined only by the power factor (pf) of the external load connected to the synchronous generator.

![Equivalent circuit of synchronous generator](image)

**Fig 1.3 Equivalent circuit of synchronous generator**

For synchronous generator the terminal voltage $V_t$ can be written as

$$V_t = E_g - jI_a X_{ar} - jI_a X_{al} - I_a R_a$$

$$V_t = E_g - jI_a X_s - I_a R_a$$

$$V_t = E_g - I_a (R_a + jX_s) = E_g - I_a Z_s$$

Where $E_g$ is the generator induced emf,

$I_a$ is the armature current,

$R_a$ is the armature resistance,

$X_{al}$ is the leakage reactance,

$X_{ar}$ is the armature reaction reactance,

$X_s$ is the synchronous reactance

$Z_s$ is the synchronous impedance

**1.7.3 Phasor Diagrams**

The complete phasor diagram of an alternator at different load conditions are shown below.
1.7.3.1 For Inductive Load

The alternator is connected with a R-L load then the current lags terminal voltage by an angle $\theta$. The phasor diagram is shown below in Fig: 1.4.

![Phasor diagram of an alternator with lagging power factor load](image1)

**Fig: 1.4**

**Phasor diagram of an alternator with lagging power factor load**

1.7.3.2 For Resistive Load

The alternator is connected with a resistive load then the current remains in same phase with the terminal voltage. The phasor diagram is shown below in Fig: 1.5.

![Phasor diagram of an alternator with unity power factor load](image2)

**Fig: 1.5** Phasor diagram of an alternator with unity power factor load
1.7.3.3 For Capacitive Load

When the terminals of the armature of alternator is connected with a R-C load then the current $I_a$ leads the terminal voltage $V_t$ by an angle $\theta$. The complete phasor diagram for leading power factor load is shown below in Fig: 1. 6.

![Fig: 1. 6 Phasor diagram of an alternator with leading power factor load](image)

$\delta$ is called load angle
$\theta$ is load power factor angle
$\psi$ is internal power factor angle

1.8 Open-circuit characteristic (OCC) of a generator

With the armature terminals open, $I_a=0$, so $E_g = V_t$. It is thus possible to construct a plot of $E_g$ or $V_t$ vs $I_f$ graph. This plot is called open-circuit characteristic (OCC) of a generator. With this characteristic, it is possible to find the internal generated voltage of the generator for any given field current.
Initially OCC follows a straight-line relation with the field current as long as the magnetic circuit of the synchronous generator does not saturate. This straight line is appropriately called the *air-gap line*. Practically due to saturation induced emf bend from the straight line.

1.9 Short Circuit Characteristics (SCC)

For getting SCC generator is rotated at rated speed with armature terminals short circuited. The field current is adjusted to 0. The armature current is measured as the field current is increased.
1.10 Armature Reaction Reactance

Armature reaction refers to the influence of the armature flux on the field flux in the air gap when the stator windings are connected across a load.

If \( F_f \) is the field mmf in the generator under no load, then the generated voltage \( E_g \) must lag \( F_f \) by 90°. Per phase armature current \( I_a \) produces armature mmf \( F_a \) which is in phase with \( I_a \). The effective mmf is \( F_r \).

\[ \overline{E_r} = \overline{E_g} + \overline{E_{ar}} \]

The armature mmf \( F_a \) will induce an emf \( E_{ar} \) in the armature winding. \( E_{ar} \) is called the armature reaction emf. This emf will lag its mmf by 90°. Hence the resultant armature voltage is the vector sum of the no-load voltage \( E_g \) and armature reaction emf \( E_{ar} \).
Fig: 1.11 Phasor diagram of an alternator at leading power factor

From the observations of the phasor diagrams for lagging and leading power factors, that the resultant mmf $F_r$ is smaller or larger depending on the power factor. As a result the terminal voltage $V_t$ is larger or smaller than the no-load induced emf when the power factor is leading or lagging.

Since the armature reaction emf $E_{ar}$ lags the armature mmf $F_a$ or $I_a$ by 90°, so it can be expressed as

$$E_{ar} = -jI_a X_{ar}$$

Where $X_{ar}$ is called **armature reaction reactance**.

1.11 Synchronous reactance

Both the armature reaction reactance and the leakage reactance are present at the same time. The two reactances are combined together and the sum is called the **Synchronous reactance** ($X_s$).

$$X_s = X_{al} + X_{ar}$$

The combined result of the Synchronous reactance and armature resistance is called **Synchronous Impedance** ($Z_s$).

$$Z_s = R_a + jX_s$$

1.12 Short Circuit Ratio (SCR)

Ratio of the field current required for the rated voltage at open circuit to the field current required for rated armature current at short circuit.

$$SCR = \frac{I_{f,oc}}{I_{f,sc}}$$
So, \( SCR = \frac{1}{X_s} \)

1.13 Load Characteristics

Consider a synchronous generator driven at constant speed and with constant excitation. On open circuit the terminal voltage \( V_t \) is the same as the open circuit e.m.f. \( E_g \). Suppose a unity-power-factor load be connected to the machine. The flow of load current produces a voltage drop \( I_aZ_s \) in the synchronous impedance, and terminal voltage \( V_t \) is reduced. Fig. 1.12 shows the phasor diagram for three types of load. It will be seen that the angle \( \sigma \) between \( E_g \) and \( V_t \) increases with load, indicating a shift of the flux across the pole faces due to cross- magnetization. The terminal voltage is obtained from the complex summation

\[
V_t + I_aZ_s = E_g
\]

\[
V_t = E_g - I_aZ_s
\]

Algebraically this can be written as-

\[
V_t = \sqrt{(E_g^2 - I_a^2X_s^2)} - I_a r_a
\]

For non-inductive load since \( r_a \) is negligible compared to \( X_s \)

\[
V_t^2 + I_a^2X_s^2 \approx E_g^2 = Constant
\]

so that the V/I curve, Fig. 1.13, is nearly an ellipse with semi-axes \( E_g \) and \( I_{sc} \). The current \( I_{sc} \) is that which flows when the load resistance is reduced to zero. The voltage \( V_t \) falls to zero also and the machine is on short-circuit with \( V_t = 0 \) and

\[
I_s = I_{sc} = E_g / Z_s \approx E_g / X_s
\]

For a lagging load of zero power-factor, diagram is given in Fig. 1.13. The voltage is given as before and since the resistance in normal machines is small compared with the synchronous reactance, the voltage is given approximately by
\[ V_t \approx E_r - I_a X_s \]

1.12 (i) Phasor diagram for different R loads

(ii)
Fig: 1.13 Variation of voltage with load at constant Excitation
which is the straight line marked for \( \cos \varphi = 0 \) lagging in Fig.1.14. A leading load of zero power factor
Fig. 1.14 will have the voltage

\[
V_t \approx E_t + I_a X_s
\]

another straight line for which, by reason of the direct magnetizing effect of leading currents, the voltage
increases with load.

Intermediate load power factors produce voltage/current characteristics resembling those in Fig: 1.13 The
voltage-drop with load (i.e. the regulation) is clearly dependent upon the power factor of the load. The
short-circuit current \( I_{sc} \) at which the load terminal voltage falls to zero may be about 150 per cent (1.5 per
unit) of normal current in large modern machines.

Fig: 1.14 Load characteristics of Alternator
1. 14 Potier Reactance

For obtaining potier reactance Zero Power Factor test is conducted by connecting the alternator to ZPF load and exciting the alternator in such way that the alternator supplies the rated current at rated voltage running at rated speed. To plot ZPF characteristics only two points are required. One point is corresponding to the zero voltage and rated current that can be obtained from scc and the other at rated voltage and rated current under zpf load. This zero power factor curve appears like OCC but shifted by a factor \( I_aX_L \) vertically and horizontally by armature reaction mmf as shown below in Fig: 1.15. Following are the steps to draw ZPF characteristics.

![Fig: 1.15](image)

By suitable tests plot OCC and SCC. Draw air gap line. Conduct ZPF test at full load for rated voltage and fix the point B. Draw the line BH with length equal to field current required to produce full load current on short circuit. Draw HD parallel to the air gap line so as to cut the OCC. Draw DE perpendicular to HB or parallel to voltage axis. Now, DE represents voltage drop \( IX_L \) and BE represents the field current required to overcome the effect of armature reaction.

Triangle BDE is called **Potier triangle** and XL is the **Potier reactance**.

1.15 Voltage Regulation

When an alternator is subjected to a varying load, the voltage at the armature terminals varies to a certain extent, and the amount of this variation determines the regulation of the machine. When the alternator is loaded the terminal voltage decreases as the drops in the machine stars increasing and hence it will always
be different than the induced emf.

Voltage regulation of an alternator is defined as the change in terminal voltage from no load to full load expressed as a percentage of rated voltage when the load at a given power factor is removed without change in speed and excitation. Or the numerical value of the regulation is defined as the percentage rise in voltage when full load at the specified power-factor is switched off with speed and field current remaining unchanged expressed as a percentage of rated voltage.

Hence regulation can be expressed as

\[
\% \text{ Regulation} = \left( \frac{E_0 - V_t}{V_t} \right) \times 100
\]

where \( E_0 \) = No-load induced emf /phase, \( V_t \) = Rated terminal voltage/phase at load

1.16 Methods of finding Voltage Regulation:

The voltage regulation of an alternator can be determined by different methods. In case of small generators it can be determined by direct loading whereas in case of large generators it cannot determined by direct loading but will be usually predetermined by different methods. Following are the different methods used for predetermination of regulation of alternators.

1. Direct loading method
2. EMF method or Synchronous impedance method
3. MMF method or Ampere turns method
4. ASA modified MMF method
5. ZPF method or Potier triangle method

All the above methods other than direct loading are valid for non-salient pole machines only. As the alternators are manufactured in large capacity direct loading of alternators is not employed for determination of regulation. Other methods can be employed for predetermination of regulation. Hence the other methods of determination of regulations will be discussed in the following sections.
1.16.1 EMF method:

This method is also known as synchronous impedance method. Here the magnetic circuit is assumed to be unsaturated. In this method the MMFs (fluxes) produced by rotor and stator are replaced by their equivalent emf, and hence called emf method.

To predetermine the regulation by this method the following informations are to be determined. Armature resistance /phase of the alternator, open circuit and short circuit characteristics of the alternator.

**Determination of synchronous impedance Z_s:**

![Diagram of OCC and SCC of alternator](image)

As the terminals of the stator are short circuited in SC test, the short circuit current is circulated against the impedance of the stator called the synchronous impedance. This impedance can be estimated form the oc and sc characteristics.

The ratio of open circuit voltage to the short circuit current at a particular field current, or at a field current responsible for circulating the rated current is called the synchronous impedance.

Synchronous impedance $Z_s = (\text{open circuit voltage per phase})/(\text{short circuit current per phase})$ for same If

Hence $Z_s = (V_{oc})/(I_{sc})$ for same If

From Fig: 1.16 synchronous impedance $Z_s = V/I_{sc}$
Armature resistance $R_a$ of the stator can be measured using Voltmeter – Ammeter method. Using synchronous impedance and armature resistance synchronous reactance and hence regulation can be calculated as follows using emf method.

\[ Z_s = \sqrt{(R_a)^2 + (X_s)^2} \]
\[ X_s = \sqrt{(Z_s)^2 - (R_a)^2} \]

Hence induced emf per phase can be found as
\[
E_g = \sqrt{\left( Vt \cos\theta + I_a R_a \right)^2 + \left( Vt \sin\theta \pm I_a X_s \right)^2}
\]

where $Vt = \text{phase voltage per phase} = Vph$, $I_a = \text{load current per phase}$

In the above expression in second term + sign is for lagging power factor and – sign is for leading power factor.

% Regulation = \[ \left( \frac{E_g - V_t}{V_t} \right) \times 100 \]

where $E_g = \text{no-load induced emf/phase}$, $V_t = \text{rated terminal voltage/phase}$

Synchronous impedance method is easy but it gives approximate results. This method gives the value of regulation which is greater (poor) than the actual value and hence this method is called pessimistic method.

The complete phasor diagram for the emf method is shown in Fig 1.18.
1.13.2 MMF method

This method is also known as amp-turns method. In this method the all the emfs produced by rotor and stator are replaced by their equivalent MMFs (fluxes), and hence called mmf method. In this method also it is assumed that the magnetic circuit is unsaturated. In this method both the reactance drops are replaced by their equivalent mmfs. Fig: 1.19 shows the complete phasor diagram for the mmf method. Similar to emf method OC and SC characteristics are used for the determination of regulation by mmf method. The details are shown in Fig: 1.19. Using the details it is possible determine the regulation at different power factors.

From the phasor diagram it can be seen that the mmf required to produce the emf $E_1 = (V + IR_a)$ is FR1. In large machines resistance drop may neglected. The mmf required to overcome the reactance drops is $(F_a + F_{al})$ as shown in phasor diagram. The mmf $(F_a + F_{al})$ can be found from SC characteristic as under SC condition both reactance drops will be present.
Following procedure can be used for determination of regulation by mmf method.

1. By conducting OC and SC test plot OCC and SCC.
2. From the OCC find the field current $I_f$ required to produce the voltage, $E_1 = (V + IR_a)$.
3. From SCC find the magnitude of field current $I_f'\approx (F_a + F_{al})$ to produce the required armature current. $F_a + F_{al}$ can also found from ZPF characteristics.
4. Draw $I_f'$ at angle $(90+\Phi)$ from $I_f$, where $\Phi$ is the phase angle of current w. r. t voltage. If current is leading, take the angle of $I_f'$ as $(90-\Phi)$.
5. Determine the resultant field current, $I_f$ and mark its magnitude on the field current axis.
6. From OCC find the voltage corresponding to $I_f$, which will be $E_0$ and hence find the regulation.

Because of the assumption of unsaturated magnetic circuit the regulation computed by this method will be less than the actual and hence this method of regulation is called optimistic method.

**1.13.3 ASA Modified MMF Method:**

ASA or modified mmf method consider saturation effect for calculation of regulation. In the mmf method the total mmf $F$ computed is based on the assumption of unsaturated magnetic circuit which is unrealistic. In order to account for the partial saturation of the magnetic circuit it must be increased by a certain amount $F_f$ which can be computed from OCC, SCC and air gap lines as explained below referring to Fig: 1.20 (i) and (ii).
If \( I_f \) is the field current required to induce the rated voltage on open circuit. Draw \( I_f \) with length equal to field current required to circulate rated current during short circuit condition at an angle \((90+\Phi)\) from \( I_f \). The resultant of \( I_f \) and \( I_f' \) gives \( I_f \) (OF2 in figure). Extend OF2 upto F so that F2F accounts for the additional field current required for accounting the effect of partial saturation of magnetic circuit. F2F is found for voltage E (refer to phasor diagram of mmf method) as shown in Fig: 1.20. Project total field current OF to the field current axis and find corresponding voltage E0 using OCC. Hence regulation can found by ASA method which is more realistic.

1.13.4 **Zero Power Factor (ZPF) method or Potier Triangle Method:**

During the operation of the alternator, resistance voltage drop \( I_aR_a \) and armature leakage reactance drop \( I_aX_L \) are actually emf quantities and the armature reaction reactance is a mmf quantity. To determine the regulation of the alternator by this method OCC, SCC and ZPF test details and characteristics are required. As explained earlier oc and SC tests are conducted and OCC and SCC are drawn. ZPF test is conducted by connecting the alternator to ZPF load and exciting the alternator in such way that the alternator supplies the rated current at rated voltage running at rated speed. To plot ZPF characteristics only two points are required. One point is corresponding to the zero voltage and rated current that can be obtained from SCC and the other at rated voltage and rated current under zpf...
load. This zero power factor curve appears like OCC but shifted by a factor $I_aX_L$ vertically and horizontally by armature reaction mmf as shown below in Fig: 1.21. Following are the steps to draw ZPF characteristics.

By suitable tests plot OCC and SCC. Draw air gap line. Conduct ZPF test at full load for rated voltage and fix the point B. Draw the line BH with length equal to field current required to produce full load current on short circuit. Draw HD parallel to the air gap line so as to cut the OCC. Draw DE perpendicular to HB or parallel to voltage axis. Now, DE represents voltage drop $IX_L$ and BE represents the field current required to overcome the effect of armature reaction.

Triangle BDE is called Potier triangle and XL is the Potier reactance. Find E from $V, IRa, IXL$ and $\Phi$. Use the expression $E = \sqrt{(V \cos \Phi + I_aRa)^2 + (V \sin \Phi + I_aXL)^2}$ to compute E. Find field current corresponding to $E$. Draw FG with magnitude equal to BE at angle $(90+\Psi)$ from field current axis, where $\Psi$ is the phase angle of current from voltage vector $E$ (internal phase angle).

The resultant field current is given by OG. Mark this length on field current axis. From OCC find the corresponding $E0$. Find the regulation.
1.14 Power angle characteristics

When the synchronous generator feeding power to the infinite bus-bar at constant terminal voltage $V_t$ as shown in single line diagram in Fig: 1.22 the phasor diagram for lagging power factor is shown if Fig: 1.23. For large size of generator armature resistance $r_a$ is negligible.

The per phase power delivered to the infinite bus is given by

$$P = V_t I_a \cos \theta$$

It is seen that $\angleoba = 90 - \theta$ and $\angleobc = 180 - (90 - \theta) = 90 + \theta$. The triangle obc reveals that

$$\frac{bc}{\sin \angleboc} = \frac{oc}{\sin \angleobc} \text{ or } \frac{X_s I_a}{\sin \delta} = \frac{E_f}{\sin (90 + \theta)}$$

or, $X_s I_a \sin (90 + \theta) = E_f \sin \delta$

$X_s I_a \cos \theta = E_f \sin \delta$

$$I_a \cos \theta = \frac{E_f}{X_s} \sin \delta$$
Substitution of value of $I_a \cos \theta$ in power equation

$$P = \frac{E_i V}{X_s} \sin \delta$$

The variation of power as derived above with respect to power-angle $\delta$ is plotted in Fig; 1.24. The power versus load angle characteristic curve has a sinusoidal shape and is usually called power-angle characteristic of the cylindrical-rotor synchronous machine. The power $P$, for generator is taken as positive and therefore, for motor as negative.

Fig; 1.24 Power angle characteristic
MODULE-II
SYNCHRONOUS GENERATOR & MOTOR

SYLLABUS/ TOPICS COVERED
Fundamental Principles of A.C. Machines: E.M.F. equation of an elementary alternator, Single & three Phase, relation between speed & frequency, factors affecting the induced e.m.f., full pitch & fractional pitch windings, winding factors, armature reaction, the rotating field leakage reactance. Concept of time phasor & space phasor. Synchronous Generator: Various types & construction, cylindrical rotor theory, phasor diagram, open circuit & short circuit characteristics, armature reaction reactance, synchronous reactance, SCR, load characteristics, potier reactance, voltage regulation, EMF method, MMF method, modified MMF method, ZPF method, power angle characteristics.
[Topics are arranged as per above sequence]
2.1 Salient pole alternators and Blondel’s Two Reaction Theory

The details of synchronous generators developed so far is applicable to only round rotor or non-salient pole alternators. In such machines the air gap is uniform throughout and hence the effect of mmf will be same whether it acts along the pole axis or the inter polar axis. Hence reactance of the stator is same throughout and hence it is called synchronous reactance. But in case salient pole machines the air gap is non uniform and it is smaller along pole axis and is larger along the inter polar axis. These axes are called direct axis or d-axis and quadrature axis or q-axis. Hence the effect of mmf when acting along direct axis will be different than that when it is acting along quadrature axis. Hence the reactance of the stator cannot be same when the mmf is acting along d – axis and q- axis. As the length of the air gap is small along direct axis reluctance of the magnetic circuit is less and the air gap along the q – axis is larger and hence the along the quadrature axis will be comparatively higher. Hence along d-axis more flux is produced than q-axis. Therefore the reactance due to armature reaction will be different along d-axis and q-axis. These reactances are,

\[ X_{ad} = \text{direct axis reactance; } X_{aq} = \text{quadrature axis reactance} \]

Hence the effect of armature reaction in the case of a salient pole synchronous machine can be taken as two components - one acting along the direct axis (coinciding with the main field pole axis) and the other acting along the quadrature axis (inter-polar region or magnetic neutral axis) and as such the mmf components of armature-reaction in a salient-pole machine cannot be considered as acting on the same magnetic circuit. Hence the effect of the armature reaction cannot be taken into account by considering only the synchronous reactance, in the case of a salient pole synchronous machine.

In fact, the direct-axis component \( F_{ad} \) acts over a magnetic circuit identical with that of the main field system and produces a comparable effect while the quadrature-axis component \( F_{aq} \) acts along the interpolar axis, resulting in an altogether smaller effect and, in addition, a flux distribution totally different from that of \( F_{ad} \) or the main field m.m.f. This explains why the application of cylindrical-rotor theory to salient-pole machines for predicting the performance gives results not conforming to the performance obtained from an actual test.
2.2 Direct-axis and Quadrature-axis Synchronous Reactances

Blondel’s two-reaction theory considers the effects of the quadrature and direct-axis components of the armature reaction separately. Neglecting saturation, their different effects are considered by assigning to each an appropriate value of armature-reaction “reactance,” respectively $x_{ad}$ and $x_{aq}$. The effects of armature resistance and true leakage reactance ($X_L$) may be treated separately, or may be added to the armature reaction coefficients on the assumption that they are the same, for either the direct-axis or quadrature-axis components of the armature current (which is almost true). Thus the combined reactance values can be expressed as,

$$X_{sd} = x_{ad} + x, \text{ and } X_{sq} = x_{aq} + x,$$

for the direct- and cross-reaction axes respectively.

In a salient-pole machine, $x_{aq}$, the quadrature-axis reactance is smaller than $x_{ad}$, the direct-axis reactance, since the flux produced by a given current component in that axis is smaller as the reluctance of the magnetic path consists mostly of the interpolar spaces. It is essential to clearly note the difference between the quadrature and direct-axis components $I_{aq}$, and $I_{ad}$ of the armature current $I_a$, and the reactive and active components $I_{aa}$ and $I_{ar}$. Although both pairs are represented by phasors in phase quadrature, the former are related to the induced emf $E_t$ while the latter are referred to the terminal voltage $V$. These phasors are clearly indicated with reference to the phasor diagram of a (salient pole) synchronous generator supplying a lagging power factor (pf) load, shown in Fig.2.1.

![Fig: 2.1 Phasor diagram of salient-pole alternator](image)
2.3 Power Angle Characteristic of Salient Pole Machine

Neglecting the armature winding resistance, the power output of the generator is given by:

\[ P = V * I_a * \cos \phi \]

This can be expressed in terms of \( \sigma \),

\[ I_a * \cos \phi = I_{aq} * \cos \sigma + I_{ad} * \sin \sigma \]

\[ V * \cos \sigma = E_0 - I_{ad} * x_{sd} \]

and \[ V * \sin \sigma = I_{aq} * x_{sd} \]

Substituting these in the expression for power, we have.

\[ P = V[(V * \sin \sigma / x_{sd}) * \cos \sigma + (E_0 - V * \cos \sigma) / x_{sd} * \sin \sigma] \]

\[ = (V * E_0 / x_{sd}) * \sin \sigma + V^2 * (x_{sd} - x_{sq}) / (2 * x_{sq} * x_{sq}) * \sin 2\sigma \]

It is clear from the above expression that the power is a little more than that for a cylindrical rotor synchronous machine, as the first term alone represents the power for a cylindrical rotor synchronous machine. A term in \( \sin 2\sigma \) is added into the power – angle characteristic of a non-salient pole synchronous machine. This also shows that it is possible to generate an emf even if the excitation \( E_0 \) is zero. However this magnitude is quite less compared with that obtained with a finite \( E_0 \). Likewise we can show that the machine develops a torque - called the reluctance torque - as this torque is developed due to the variation of the reluctance in the magnetic circuit even if the excitation \( E_0 \) is zero. Fig: 2.2 shows the typical power angle characteristic of a salient pole alternator.

![Fig: 2.2](image-url)
2.4 Slip Test

From this test the values of $X_d$ and $X_q$ are determined by applying a balance reduced external voltage (say, $V$ volts, around 25% of rated value) to the armature. The field winding remains unexcited. The machine is run at a speed a little less than the synchronous speed (the slip being less than 1%) using a prime mover (or motor). Connection diagram is shown in circuit diagram.

Due to voltage $V$ applied to the stator terminal a current $I$ will flow causing a stator mmf. This stator mmf moves slowly relative to the poles and induced an emf in the field circuit in a similar fashion to that of rotor in an induction motor at slip frequency. The effect will be that the stator mmf will moves slowly relative to the poles.

The physical poles and the armature-reaction mmf are alternately in phase and out, the change occurring at slip frequency. When the axis of the pole and the axis of the armature reaction mmf wave coincide, the armature mmf acts through the field magnetic circuit. Since the applied voltage is constant, the air-gap flux would be constant. When crest of the rotating armature mmf is in line with the field-pole axis, minimum air-gap offers minimum reluctance thus the current required in armature for the establishment of constant air-gap flux must be minimum. Constant applied voltage minus the minimum impedance voltage drop in the armature terminal gives maximum armature terminal

Fig: 2.3
voltage. Thus the d-axis synchronous reactance is given by

\[ X_d = \frac{\text{Maximum armature terminal voltage per phase}}{\text{Minimum armature current per phase}} \]

Similarly

\[ X_q = \frac{\text{Minimum armature terminal voltage per phase}}{\text{Maximum armature current per phase}} \]

### 2.5 Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as **synchronizing**. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system, to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to **infinite** bus-bars.

For proper synchronization of alternators, the following four conditions must be satisfied

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.
2. The speed of the incoming machine must be such that its frequency \(= \frac{PN}{60}\) equals bus-bar frequency.
3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage.
4. The phase angle between identical phases must be zero.

It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

Condition (1) is indicated by a voltmeter, conditions (2), (3) and (4) are indicated by synchronizing lamps or a synchronoscope.

The synchronizing lamp method consists of 3 lamps connected between the phases of the running 3-ph generator and the incoming generator as shown in Fig: 2.4.
In three phase alternators, it is necessary to synchronize one phase only, the other two phases be will then synchronized automatically. However, first it is necessary that the incoming alternator is correctly ‘phased out’ i.e. the phases are connected in the proper order of $R, Y, B$ not $R, B, Y$ etc. Lamp $L_1$ is connected between $R$ and $R'$, $L_2$ between $Y$ and $B'$ (not $Y$ and $Y'$) and $L_3$ between $B$ and $Y'$ (and not $B$ and $B'$) as shown in Fig: 2.5.

Fig: 2.4

Fig: 2.5
Two set of star vectors will rotate at unequal speeds if the frequencies of the two are different. If the incoming alternator is running faster, then voltage star \( R' Y' B' \) appear to rotate anticlockwise with respect to the bus-bar voltage star \( RYB \) at a speed corresponding to the difference between their frequencies. With reference to Fig: 2.6, it is seen that voltage across \( L_1 \) is \( RR' \) to be increasing from zero, and that across \( L_2 \) is \( YB' \) which is decreasing, having just passed through its maximum, and that across \( L_3 \) \( BY' \) which is increasing and approaching its maximum. Hence the lamps will light up one after the other in the order 2, 3, 1,2,3,1 or 1, 2, 3. If the incoming alternator is running slower, then the sequence of light up will be 1, 3, 2. Synchronization is done at the moment the uncrossed lamp \( L_1 \) is in the middle of the dark period and other two lamps are equally bright. Hence this method of synchronization is known as two bright one dark lamp method.

It should be noted that synchronization by lamps is not quite accurate, because to a large extent, it depends on the sense of correct judgment of the operator. Hence, to eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope as shown in Fig: 2.7. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.
2.5.1 Synchronizing Current:

If two alternators generating exactly the same emf are perfectly synchronized, there is no resultant emf acting on the local circuit consisting of their two armatures connected in parallel. No current circulates between the two and no power is transferred from one to the other. Under this condition emf of alternator 1, i.e. E1 is equal to and in phase opposition to emf of alternator 2, i.e. E2 as shown in the Figure. There is, apparently, no force tending to keep them in synchronism, but as soon as the conditions are disturbed a synchronizing force is developed, tending to keep the whole system stable. Suppose one alternator falls behind a little in phase by an angle θ. The two alternator emfs now produce a resultant voltage and this acts on the local circuit consisting of the two armature windings and the joining connections. In alternators, the synchronous reactance is large compared with the resistance, so that the resultant circulating current Is is very nearly in quadrature with the resultant emf Er acting on the circuit. Figure represents a single phase case, where E1 and E2 represent the two induced emfs, the latter having fallen back slightly in phase. The resultant emf, Er, is almost in quadrature with both the emfs, and gives rise to a current, Is, lagging behind Er by an angle approximating to a right angle. It is, thus, seen that E1 and Is are almost in phase. The first alternator is generating a power $E1 \, Is \, \cos \Phi1$, which is positive, while the second one is generating a power $E2 \, Is \, \cos \Phi2$, which is negative, since $\cos \Phi2$ is negative. In other words, the first alternator is supplying...
the second with power, the difference between the two amounts of power represents the copper losses occasioned by the current $I_s$ flowing through the circuit which possesses resistance. This power output of the first alternator tends to retard it, while the power input to the second one tends to accelerate it till such a time that $E_1$ and $E_2$ are again in phase opposition and the machines once again work in perfect synchronism. So, the action helps to keep both machines in stable synchronism. The current, $I_s$, is called the synchronizing current.

![Fig: 2.7](image)

### 2.5.2 Effect of Change of Excitation:
A change in the excitation of an alternator running in parallel with other affects only its KVA output; it does not affect the KW output. A change in the excitation, thus, affects only the power factor of its output. Let two similar alternators of the same rating be operating in parallel, receiving equal power inputs from their prime movers. Neglecting losses, their kW outputs are therefore equal. If their excitations are the same, they induce the same emf, and since they are in parallel their terminal voltages are also the same. When delivering a total load of $I$ amperes at a power-factor of $\cos \phi$, each alternator delivers half the total current and $I_1 = I_2 = I/2$.

![Fig: 2.8](image)

Since their induced emfs are the same, there is no resultant emf acting around the local circuit formed by their two armature windings, so that the synchronizing current, $I_s$, is zero. Since the armature
resistance is neglected, the vector difference between $E_1 = E_2$ and $V$ is equal to, $I_1X_{S1} = I_2X_{S2}$, this vector leading the current $I$ by $90^\circ$, where $X_{S1}$ and $X_{S2}$ are the synchronous reactances of the two alternators respectively.

Now consider the effect of reducing the excitation of the second alternator. $E_2$ is therefore reduced as shown in Figure. This reduces the terminal voltage slightly, so let the excitation of the first alternator be increased so as to bring the terminal voltage back to its original value. Since the two alternator inputs are unchanged and losses are neglected, the two kW outputs are the same as before. The current $I_2$ is changed due to the change in $E_2$, but the active components of both $I_1$ and $I_2$ remain unaltered. It can be observed that there is a small change in the load angles of the two alternators, this angle being slightly increased in the case of the weakly excited alternator and slightly decreased in the case of the strongly excited alternator. It can also be observed that $I_1 + I_2 = I$, the total load current.

### 2.5.3 Effect of Change of Input Torque

The amount of power output delivered by an alternator running in parallel with others is governed solely by the power input received from its prime mover. If two alternators only are operating in parallel the increase in power input may be accompanied by a minute increase in their speeds, causing a proportional rise in frequency. This can be corrected by reducing the power input to the other alternator, until the frequency is brought back to its original value. In practice, when load is transferred from one alternator to another, the power input to the alternator required to take additional load is increased, the power input to the other alternator being simultaneously decreased. In this way, the change in power output can be effected without measurable change in the frequency. The effect of increasing the input to one prime mover is, thus, seen to make its alternator take an increased share of the load, the other being relieved to a corresponding extent. The final power-factors are also altered, since the ratio of the reactive components of the load has also been changed. The power-factors of the two alternators can be brought back to their original values, if desired, by adjusting the excitations of alternators.
2.5.4 Load Sharing

When several alternators are required to run in parallel, it probably happens that their rated outputs differ. In such cases it is usual to divide the total load between them in such a way that each alternator takes the load in the same proportion of its rated load in total rated outputs. The total load is not divided equally. Alternatively, it may be desired to run one large alternator permanently on full load, the fluctuations in load being borne by one or more of the others. If the alternators are sharing the load equally the power triangles are as shown in Fig: 2.9.

2.5.5 Sharing of load when two alternators are in parallel

Consider two alternators with identical speed load characteristics connected in parallel as shown in Fig: 2.10.
Let $E_1$, $E_2$ be the induced emf per phase,
$Z_1$, $Z_2$ be the impedances per phase,
$I_1$, $I_2$ be the current supplied by each machine per phase,
$Z$ be the load impedance per phase,
$V$ be the terminal voltage per phase.

From the circuit we have $V = E_1 - I_1Z_1 = E_2 - I_2Z_2$
and hence, $I_1 = E_1 - V/Z_1$ and $I_2 = E_2 - V/Z_2$

and also $V = (I_1 + I_2)Z = IZ$
solving above equations

$$I_1 = \left[ (E_1- E_2) Z + E_1 Z_2 \right] / \left[ Z( Z_1 + Z_2) + Z_1Z_2 \right]$$

$$I_2 = \left[ (E_2- E_1) Z + E_2 Z_1 \right] / \left[ Z( Z_1 + Z_2) + Z_1Z_2 \right]$$

The total current $I = I_1 + I_2 = \left[ E_1Z_2 + E_2Z_1 \right] / \left[ Z( Z_1 + Z_2) + Z_1Z_2 \right]$

And the circulating current or synchronizing current $I_s = (E_1 - E_2) / (Z_1 + Z_2)$

### 2.5.6 Prime-mover Governor Characteristic

The transfer of active power between alternators in parallel is accomplished by adjustment of the no-load speed setting of the respective prime-mover governors, and the transfer of reactive power is accomplished by adjustment of the respective field rheostats or voltage regulators. A typical prime-mover governor characteristic, shown in Fig. 2.11, is a plot of prime-mover speed (or generator frequency) vs. active power. Although usually drawn as a straight line, the actual characteristic has a slight curve. The drooping characteristic shown in the figure provides inherent stability of operation when paralleled with other machines. Machines with zero droop, called isochronous machines, are inherently unstable when operated in parallel; they are subject to unexpected load swings, unless electrically controlled with solid-state regulators.

The no-load speed setting (and hence the no-load frequency setting) of a synchronous generator can be changed by remote control from the generator panel by using a remote-control switch. The switch actuates a servomotor that repositions the no-load speed setting of the governor, raising or lowering the characteristic without changing its slope. Curves for different no-load speed settings are shown with broken lines in Figure 2.11.

**Governor Speed Regulation**

Governor speed regulation (GSR) is defined as:

$$GSR = \frac{n_{nl} - n_{rated}}{n_{rated}} = \frac{f_{nl} - f_{rated}}{f_{rated}}$$

Where, $n_{rated} = \text{rated speed (r/min)}$
$n_{nl} = \text{no-load speed (r/min)}$
$f_{rated} = \text{rated frequency (Hz)}$ & $f_{nl} = \text{no-load frequency (Hz)}$
Governor Droop

Governor droop (GD) or droop rate is defined as the ratio of the change in frequency to the corresponding change in active power:

\[
GD = \frac{\Delta f}{\Delta P} = \frac{f_{nl} - f_{rated}}{P_{rated}}
\]

Where,

- \(f_{rated}\) = rated frequency (Hz)
- \(f_{nl}\) = no-load frequency (Hz)
- \(P_{rated}\) = rated active power (kW)

2.6 Sudden Short Circuit of a Synchronous Generator

It may be possible in practice that the alternator running with full excitation may undergo a sudden short circuit because of the abnormal conditions. Due to sudden short circuit of alternator, large mechanical forces are developed which may not be sustained by the alternator. These forces are proportional to square of the current value, hence large pressure is built up between adjacent stator conductors.

The short circuit transients in a synchronous machine is a complicated phenomenon due to number of circuits coupled to each other are involved. When a synchronous generator undergoes short circuit, it has a characteristics time varying behaviour. During short circuit, flux per pole dynamically changes. Thus the transients are seen in the field and damper windings. The alternator can be represented by an equivalent circuit wherein the reactance is seen to be changed from subtransient reactance to final steady state synchronous reactance.

When alternator undergoes a short circuit number of events take place which depends on various factors such as the instant in the cycle at which short circuit occurs, whether the machine is loaded or not, what is the excitation provided, how many phases are involved, whether it is occurring near to machine terminals or away from it and on the constructional features of the machine. Hence the
evaluation of sudden short circuit current for the given conditions is complex and to some extent empirical process depending on values of resistance, self and mutual inductances which themselves are variable and difficult to assess.

After the moment of short circuit, the time period followed by it can be divided into three periods. The first one is very short period of one or two cycles the conditions of which are dependent on the flux linkages between stator and rotor during short circuit. The second interval is longer one which is nothing but transient decay of short circuit current which is affected by damping and rise of armature reaction. The final period is nothing but the steady state short circuit before which the generator is normally open circuited [see Fig: 2.12].

![Fig: 2.12](image)

**Constant Flux Linkage Theorem**

The behavior shown by the alternator just after short circuit can be understood by the use of constant linkages theorem. If a closed circuit with resistance \( r \) and inductance \( L \) is considered without a source then the equation obtained using KVL will be \( ri + L \frac{di}{dt} = 0 \). If \( r \) is very very small then \( L \frac{di}{dt} = 0 \) or \( \frac{d}{dt} (LI) = 0 \) This shows that the flux linkages \( Li \) remain constant. In generator also the effective inductance of stator and rotor windings is large compared to the resistance which can be neglected for first few cycles. The rotor circuit is closed through exciter while stator is closed by short circuit. Thus the flux linking with either winding must remain constant irrespective of the rotation.

**Analysis of RL Series Circuit**

Similar to theorem of constant linkages let us consider a series R-L circuit excited by a voltage source which is sinusoidal for further understanding of short circuit. The circuit diagram is as shown in the Fig: 2.13
Let at the instant \( t = t_1 \), the sinusoidal voltage \( V \sin \omega t \) is applied to series R-L circuit.

Applying KVL,
\[
Ri + L\frac{di}{dt} = V \sin \omega t
\]

For the above equation the complementary function of the solution is
\[
i_{CF} = K_1 e^{(R/L)t}
\]

For obtaining the particular solution let the trial solution be
\[
i_{PI} = A \cos \omega t + B \sin \omega t
\]

where A and B are undetermined coefficients.

\[
\begin{align*}
i_{PI}' & = -A \sin \omega t(\omega) + B \cos \omega t(\omega) \\
i_{PI}' & = -A \omega \sin \omega t + B \omega \cos \omega t
\end{align*}
\]

Substituting trial solution and its derivative in equation (1)
\[
R(A \cos \omega t + B \sin \omega t) + L (-A \sin \omega t + B \omega \cos \omega t) = V \sin \omega t
\]

Comparing coefficient of like terms

\[
\begin{align*}
RB - LA\omega & = V \\
RA + \omega LB & = 0
\end{align*}
\]

Solving above equations we get

\[
\begin{align*}
A & = -\frac{\omega B}{R^2 + \omega^2 L^2} \\
B & = \frac{V R}{R^2 + \omega^2 L^2}
\end{align*}
\]

The particular solution is therefore given by,

\[
i_{PI} = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \left[ \sin \left( \omega t - \tan^{-1} \frac{\omega L}{R} \right) \right]
\]

To find the value of \( K_1 \) let us use initial conditions i.e.

\[
\begin{align*}
K_1 & = e^{-(R/L)t_1} \left[ -\frac{V}{\sqrt{R^2 + (\omega L)^2}} \right] \sin \left[ \omega t_1 - \tan^{-1} \frac{\omega L}{R} \right]
\end{align*}
\]
Hence the complete solution is given as,

\[
\begin{align*}
    i &= e^{-(R/L) t} \left[ \frac{-V}{\sqrt{R^2 + (\omega L)^2}} \sin \left( \omega t_1 - \tan^{-1} \frac{\omega L}{R} \right) \right] \\
    &= e^{-(R/L) t} + \frac{V}{\sqrt{R^2 + (\omega L)^2}} \left[ \sin \left( \omega t - \tan^{-1} \frac{\omega L}{R} \right) \right] \\
    \therefore \quad i &= \frac{V}{\sqrt{R^2 + (\omega L)^2}} \sin \left( \omega t - \tan^{-1} \frac{\omega L}{R} \right) \\
    &\quad - \frac{V}{\sqrt{R^2 + (\omega L)^2}} \sin \left( \omega t_1 - \tan^{-1} \frac{\omega L}{R} \right) e^{-(R/L)(t-t_1)}
\end{align*}
\]

Let

\[ Z = \sqrt{R^2 + (\omega L)^2} \]

\[ \phi = \tan^{-1} \left( \frac{\omega L}{R} \right) \]

Substituting in above equation,

\[
i = \frac{V}{Z} \sin (\omega t - \phi) - \frac{V}{Z} \sin (\omega t_1 - \phi) e^{-(R/L)(t-t_1)}
\]

The corresponding waveforms are as shown in the Fig: 2.14.

\[ \text{Fig: 2.14} \]

The first term steady state current \((i_s)\) while the second term represents transient current \((i_t)\). If the
voltage is switched at $t = t_1$ when it is zero, the transient term is having the greatest value. The resultant current is zero having complete asymmetry. The approximate current in this case reaches $2im$ which is known as doubling effect compared to the switching of voltage at the instant when voltage is at its maximum instead of zero. This shows that the current flowing in the circuit changes its waveform if the instant at which voltage is applied to the circuit, is changed. The same thing is applicable in case of generator undergoing short circuit.

**Short Circuit Phenomenon**

Consider a two pole elementary single phase alternator with concentrated stator winding as shown in Fig: 2.15.

![Fig: 2.15](image1)

The corresponding waveforms for stator and rotor currents are shown in the Fig: 2.15.

Let short circuit occurs at position of rotor shown in Fig: 2.15(a), when there are no stator linkages. After 1/4 Rev as shown Fig: 2.15(b), it tends to establish full normal linkage in stator winding. The stator opposes this by a current in the shown direction as to force the flux in the leakage path. The rotor current must increase to maintain its flux constant. It reduces to normal at position (c) where stator current is again reduces to zero. The waveform of stator current and field current shown in the Fig: 2.16, changes totally if the position of rotor at the instant of short circuit is different. Thus the short circuit current is a function of of relative position of stator and rotor.

![Fig: 2.16](image2)
Using the theorem of constant linkages a three phase short circuit can also be studied. After the instant of short circuit the flux linking with the stator will not change. A stationary image of main pole flux is produced in the stator. Thus a D.C. component of current is carried by each phase. The magnitude of D.C. component of current is different for each phase as the instant on the voltage wave at which short circuit occurs is different for each phase. The rotor tries to maintain its own poles. The rotor current is normal each time when rotor poles occupy the position same as that during short circuit and the current in the stator will be zero if the machine is previously unloaded. After one half cycle from this position the stator and rotor poles are again coincident but the poles are opposite. To maintain the flux linkages constant, the current in rotor reaches to its peak value.

The stationary field produced by poles on the stator induces a normal frequency emf in the rotor. Thus the rotor current is fluctuating whose resultant a.c. component develops fundamental frequency flux which rotates and again produces in the stator winding double frequency or second harmonic currents. Thus the waveform of transient current consists of fundamental, a.c. and second harmonic components of currents.

Thus whenever short circuit occurs in three phase generator then the stator currents are distorted from pure sine wave and are similar to those obtained when an alternating voltage is suddenly applied to series R-L circuit.
2.6.1 Stator Currents during Short Circuit

If a generator having negligible resistance, excited and running on no load is suddenly undergoing short circuit at its terminals, then the emf induced in the stator winding is used to circulate short circuit current through it. Initially the reactance to be taken into consideration is not the synchronous reactance of the machine. The effect of armature flux (reaction) is to reduce the main field flux. But the flux linking with stator and rotor cannot change instantaneously because of the induction associated with the windings. Thus at the short circuit instant, the armature reaction is ineffective. It will not reduce the main flux. Thus the synchronous reactance will not come into picture at the moment of short circuit. The only limiting factor for short circuit current at this instant is the leakage reactance. After some time from the instant of short circuit, the armature reaction slowly shows its
effect and the alternator then reaches to steady state. Thus the short circuit current reaches to high value for some time and then settles to steady value.

It can be seen that during the initial instant of short circuit is dependent on induced emf and leakage reactance which is similar to the case which we have considered previously of voltage source suddenly applied to series R-L circuit. The instant in the cycle at which short occurs also affects the short circuit current. Near zero e.m.f. (or voltage) it has doubling effect. The expressions that we have derived are applicable only during initial conditions of short circuit as the induced emf also reduces after some time because of increased armature reaction.

The short circuit currents in the three phases during short circuit are as shown in the Fig: 2.17.

2.7 Transient and Subtransient Reactance of Alternators

To understand the behavior of an alternator under transient conditions, the armature and field resistance is assumed to be negligibly small. Thus, constant flux linkage theorem can be applied. As per this theorem, in purely inductive circuit, the total flux linkage cannot be changed instantaneously at the time of any disturbance. Now, if all the three phases of unloaded alternator with normal excitation are suddenly short circuited there will be short- circuit current flows in the armature. As the resistance is assumed to be zero, this current will lag behind the voltage by 90° and the m.m.f. produced by this current will be along the d-axis. First conclusion is that this current will be affected by d-axis parameters Xd , Xd’ and Xd” only.

Further, there will be demagnetizing effect of this current, but as the flux linkage with field cannot change the effect of demagnetizing armature m.m.f. must be counterbalanced by a proportional increase in the field current. This additional induced component of field current gives rise to greater excitation under transient state and results in more short circuits as compared to the steady state short circuit current.

If field poles are provided with damper bars, then at the instant of three phase short circuit, the demagnetizing armature m.m.f. induces currents in damper bars, which, in turn, produces field in the same direction as the main field and hence at this instant, the excitation further increases and gives rise to further increase in short circuit armature current. This is for a very short duration, normally 3 to 4 cycles and this period is known as sub-transient period. Since the field voltage is constant, there is no additional voltage to sustain these increased excitations during sub transient or transient period. Consequently the effect of increased field current decreases with a time constant determined by the field and armature parameters and accordingly the short circuit armature current also decays with the same time constant.

In the Fig: 2.18 a symmetrical wave from for armature short circuit current of phase – A is shown. The D.C. component is zero in this phase.

The reactances offered by the machine during sub transient period are known as sub transient reactances. Along the direct axis, it is direct axis sub transient reactance, X"d and along the quadrature axis, it is quadrature axis sub-transient reactance, X”q. As these reactances are due to the fact that flux linkages in field circuit during sudden disturbance remain constant, the sub transient reactances Xd” and Xq” can also be defined as below:
Fig: 2.18

**Direct axis sub-transient reactance \( X''_d \)**

The field structure is assumed to have damper bars on salient poles. The field winding is initially unexcited and is short-circuited so that field flux-linkage is zero. Armature currents now are suddenly applied in such time phase that the peak of varying armature m.m.f. wave is in direct axis. As per constant flux linkage theorem, since the flux linkage before this is zero. Hence, it remains zero just after the application of armature m.m.f. wave and in order to maintain the flux linkages zero, current are induced in damper bars, additional rotor circuit formed by pole-body etc. and the field winding. The field of the varying armature m.m.f. is forced to drive the flux through the leakage paths mainly in air as shown in Fig: 2.19 (a).

Fig: 2.19 (a)
The armature flux linkage per ampere under these conditions is known as direct axis sub transient inductance \(L_d\).

**Quadrature axis sub transient reactance, \(X_q\)**

This also is defined in a manner similar to \(X_d\), but in this case, armature currents are applied in such time phase that the peak of varying armature m.m.f. wave is along the quadrature axis. The damper bars in the quadrature axis force the field of the varying armature m.m.f. to follow the leakage path as shown Fig: 2.19 (b).

As before, the flux linkage with q-axis damper bars must remain constant i.e. zero before and after the sudden application of armature m.m.f. Under these conditions, the armature flux linkages per ampere is known as q-axis sub transient inductance \(L_q\) and \(X_q = \omega L_q\).

To determine \(X_d\) and \(X_q\), the above mentioned conditions are created there. Two phases of the three phase alternator are connected in series and the combination is connected to a low voltage single phase supply. Field winding is short circuited. The rotor is rotated and brought along the d-axis once. \(X_d\) can be calculated from the armature current and voltage per phase of armature in this position. Next, rotor is brought along the q-axis position and \(X_q\) is determined.
2.8 Synchronous Motors

It may be recalled that a D.C. generator can be run as a D.C. motor. In same way, an alternator may operate as a motor by connecting its armature winding to a 3-phase supply. It is then called a synchronous motor. As the name implies, a synchronous motor runs at synchronous speed (\(N_s = 120f/P\)) i.e., in synchronism with the revolving field produced by the 3-phase supply. The speed of rotation is, therefore, tied to the frequency of the source. Since the frequency is fixed, the motor speed stays constant irrespective of the load or voltage of 3-phase supply. However, synchronous motors are not used so much because they run at constant speed (i.e., synchronous speed) but it found very useful applications because they possess other unique electrical properties.

*General Physical Concept*

Let assume that the armature winding (laid out in the stator) of a 3-phase synchronous machine is connected to a suitable balanced 3-phase source and the field winding to a D.C. source of rated voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e. at any location in the stator there will be a North pole at some instant of time and it will become a South pole after a time period corresponding to half a cycle. (after a time = \(1/2f\), where \(f\) = frequency of the supply). Assume that the stationary South pole in the rotor is aligned with the North pole in the stator moving in clockwise direction at a particular instant of time, as shown in Figure below. These two poles get attracted and try to maintain this alignment (as per Lenz’s law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However, the rotor cannot move instantaneously due to its mechanical inertia, and so it needs some time to move. In the meantime, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South pole. So the force of attraction will no longer be present and instead the like poles experience a force of repulsion as shown in Fig: 2.20 & Fig: 2.21. In other words, the conditions are now suitable for the production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole would again change to North pole after a time of \(1/2f\). Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to \(1/2f\) seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.
On the contrary if the rotor is brought to near synchronous speed by some external device say a small motor mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the motor is disconnected. Thus the synchronous rotor cannot start rotating on its own when the rotor and stator are supplied with rated voltage and frequency and hence the synchronous motor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to run at its synchronous speed.

2.9 Construction
A synchronous motor is a machine that operates at synchronous speed and converts electrical energy into mechanical energy. It is fundamentally an alternator operated as a motor. Like an alternator, a
synchronous motor has the following two parts:
(i) a stator which houses 3-phase armature winding in the slots of the stator core and receives power from a 3-phase supply [See (Fig: 2.22 )].
(ii) a rotor that has a set of salient poles excited by direct current to form alternate N and S poles. The exciting coils are connected in series to two slip rings and direct current is fed into the winding from an external exciter mounted on the rotor shaft. The stator is wound for the same number of poles as the rotor poles. As in the case of an induction motor, the number of poles determines the synchronous speed of the motor,

\[ N_s = \frac{120f}{P} \]

Where,
- \( f \) = frequency of supply in Hz
- \( P \) = number of poles

An important drawback of a synchronous motor is that it is not self-starting and auxiliary means have to be used for starting it.

### 2.10 Operating Principle

The fact that a synchronous motor has no starting torque can be easily explained.

(i) Consider a 3-phase synchronous motor having two rotor poles NR and SR. Then the stator will also be wound for two poles Ns and Ss. The motor has direct voltage applied to the rotor winding and a 3-phase voltage applied to the stator winding. The stator winding produces a rotating field which revolves round the stator at synchronous speed \( N_s = \frac{120f}{P} \). The direct (or zero frequency) current sets up a two-pole field which is stationary so long as the rotor is not turning. Thus, we have a situation in which there exists a pair of revolving armature poles (i.e., \( N_S - S_S \)) and a pair of stationary rotor poles (i.e., \( N_R - S_R \)).

(ii) Suppose at any instant, the stator poles are at positions A and B as shown in Fig: 2.22. It is clear that poles NS and NR repel each other and so do the poles SS and SR. Therefore, the rotor tends to move in the anticlockwise direction. After a period of half-cycle (or \( \frac{1}{2} f = 1/100 \) second), the polarities of the stator poles are reversed but the polarities of the rotor poles remain the same as shown in Fig: 2.22. Now SS and NR attract each other and so do NS and SR. Therefore, the rotor tends to move in the clockwise direction. Since the stator poles change their polarities rapidly, they tend to pull the rotor first in one direction and then after a period of half-cycle in the other. Due to high inertia of the rotor, the motor fails to start. Hence, a synchronous motor has no self-starting torque i.e., a synchronous motor cannot start by itself.
2.11 Equivalent Circuit

Unlike the induction motor, the synchronous motor is connected to two electrical systems; a d.c. source at the rotor terminals and an a.c. system at the stator terminals.

1. Under normal conditions of synchronous motor operation, no voltage is induced in the rotor by the stator field because the rotor winding is rotating at the same speed as the stator field. Only the impressed direct current is present in the rotor winding and ohmic resistance of this winding is the only opposition to it as shown in Fig: 2.23 (i).

2. In the stator winding, two effects are to be considered, the effect of stator field on the stator winding and the effect of the rotor field cutting the stator conductors at synchronous speed.

(i) The effect of stator field on the stator (or armature) conductors is accounted for by including an inductive reactance in the armature winding. This is called synchronous reactance Xs. A resistance Ra must be considered to be in series with this reactance to account for the copper losses in the stator or armature winding as shown in Fig: 2.23 (i). This resistance combines with synchronous reactance and gives the synchronous impedance of the machine.
(ii) The second effect is that a voltage is generated in the stator winding by the synchronously-revolving field of the rotor as shown in Fig: 2.23 (i). This generated e.m.f. $E_b$ is known as back e.m.f. and opposes the stator voltage $V$. The magnitude of $E_b$ depends upon rotor speed and rotor flux $\phi$ per pole. Since rotor speed is constant; the value of $E_b$ depends upon the rotor flux per pole i.e. exciting rotor current $I_r$.

Fig: 2.23 (i) shows the schematic diagram for one phase of a star-connected synchronous motor while Fig: 2.23 (ii) shows its equivalent circuit. Referring to the equivalent circuit in Fig: 2.23 (ii).

Net voltage/phase in stator winding is

$$E_r = V - E_b$$

Armature current/phase,

$$I_a = \frac{E_s}{Z_s}$$

$$Z_s = \sqrt{R_a^2 + X_s^2}$$

This equivalent circuit helps considerably in understanding the operation of a synchronous motor.

A synchronous motor is said to be normally excited if the field excitation is such that $E_b = V$. If the field excitation is such that $E_b < V$, the motor is said to be under-excited. The motor is said to be over-excited if the field excitation is such that $E_b > V$. As we shall see, for both normal and under excitation, the motor has lagging power factor. However, for over-excitation, the motor has leading power factor.

### 2.12 Phasor Diagram

Fig: 2.24 shows the phasor diagrams for different field excitations at constant load. Fig: 2.24 (i) shows the phasor diagram for normal excitation ($E_b = V$), whereas Fig: 2.24 (ii) shows the phasor diagram for under-excitation. In both cases, the motor has lagging power factor. Fig: 2.24 (iii) shows the phasor diagram when field excitation is adjusted for unity p.f. operation. Under this condition, the resultant voltage $E_r$ and, therefore, the stator current $I_a$ are minimum. When the motor is overexcited, it has leading power factor as shown in Fig: 2.24 (iv). The following points may be remembered:

(i) For a given load, the power factor is governed by the field excitation; a weak field produces the lagging armature current and a strong field produces a leading armature current.

(ii) The armature current ($I_a$) is minimum at unity p.f and increases as the p.f. becomes less either leading or lagging.

![Fig: 2.24](image-url)
2.13 Torque and Power Relations

**Motor Torque**
Gross torque, $T = 9.55 \frac{P_m}{N_s} \text{ N-M}$ where $P_m$ = Gross motor output in watts = $E_b I_a \cos(d - \phi)$
$N_s$ = Synchronous speed in r.p.m.
Shaft torque, $T_{sh} = 9.55 \frac{P_{sh_{out}}}{N_s} \text{ N-M}$
It may be seen that torque is directly proportional to the mechanical power because rotor speed (i.e., $N_s$) is fixed.

**Mechanical Power Developed**
Neglecting the armature resistance Fig: 2.25 shows the phasor diagram of an under-excited synchronous motor driving a mechanical load. Since armature resistance $R_a$ is assumed zero. $\tan \theta = \frac{X_s}{R_a} = \infty$ and hence $\theta = 90^\circ$.
Input power/phase = $V I_a \cos \phi$
Since $R_a$ is assumed zero, stator Cu loss $(I_{Ra})^2$ will be zero. Hence input power is equal to the mechanical power $P_m$ developed by the motor.
Mechanical power developed/ phase, $P_m = V I_a \cos \phi$, referring to the phasor diagram in Fig: 2.25.

Fig: 2.25
2.14 V-Curves and Inverted V-Curves

It is clear from above discussion that if excitation is varied from very low (under excitation) to very high (over excitation) value, then current \( I_a \) decreases, becomes minimum at unity p.f. and then again increases. But initial lagging current becomes unity and then becomes leading in nature. This can be shown as in the Fig: 2.26.

![V-Curves and Inverted V-Curves](image)

Excitation can be increased by increasing the field current passing through the field winding of synchronous motor. If graph of armature current drawn by the motor \( I_a \) against field current \( I_f \) is
plotted, then its shape looks like an English alphabet V. If such graphs are obtained at various load conditions we get family of curves, all looking like V. Such curves are called V-curves of synchronous motor. These are shown in the Fig: 2.27 (a).

As against this, if the power factor (cos Φ) is plotted against field current (I_f), then the shape of the graph looks like an inverted V. Such curves obtained by plotting p.f. against If, at various load conditions are called Inverted V-curves of synchronous motor. These curves are shown in the Fig: 2.27 (b).

2.15 Effect of Changing Field Excitation at Constant Load

In a d.c. motor, the armature current I_a is determined by dividing the difference between V and Eb by the armature resistance R_a. Similarly, in a synchronous motor, the stator current (I_a) is determined by dividing voltage-phasor resultant (E_r) between V and Eb by the synchronous impedance Z_s. One of the most important features of a synchronous motor is that by changing the field excitation, it can be made to operate from lagging to leading power factor. Consider a synchronous motor having a fixed supply voltage and driving a constant mechanical load. Since the mechanical load as well as the speed is constant, the power input to the motor (=3 V*I_a *cos φ) is also constant. This means that the in-phase component I_a cos φ drawn from the supply will remain constant. If the field excitation is changed, back e.m.f Eb also changes. This results in the change of phase position of I_a w.r.t. V and hence the power factor cos φ of the motor changes. Fig: 2.28 shows the phasor diagram of the synchronous motor for different values of field excitation. Note that extremities of current phasor I_a lie on the straight line AB.

(i) Under excitation

The motor is said to be under-excited if the field excitation is such that Eb < V. Under such conditions, the current I_a lags behind V so that motor power factor is lagging as shown in Fig: 2.28 (i). This can be easily explained. Since Eb < V, the net voltage E_r is decreased and turns clockwise. As angle δ =
90°) between Er and Ia is constant, therefore, phasor Ia also turns clockwise i.e., current Ia lags behind the supply voltage. Consequently, the motor has a lagging power factor.

(ii) Normal excitation

The motor is said to be normally excited if the field excitation is such that Eb = V. This is shown in Fig: 2.28 (ii). Note that the effect of increasing excitation (i.e., increasing Eb) is to turn the phasor Er and hence Ia in the anti-clockwise direction i.e., Ia phasor has come closer to phasor V. Therefore, p.f. increases though still lagging. Since input power (=3 V*Ia *cos φ) is unchanged, the stator current Ia must decrease with increase in p.f.

Suppose the field excitation is increased until the current Ia is in phase with the applied voltage V, making the p.f. of the synchronous motor unity [See Fig: 2.28 (iii)]. For a given load, at unity p.f. the resultant Er and, therefore, Ia are minimum.

(iii) Over excitation

The motor is said to be overexcited if the field excitation is such that Eb > V. Under-such conditions, current Ia leads V and the motor power factor is leading as shown in Fig: 2.28 (iv). Note that Er and hence Ia further turn anti-clockwise from the normal excitation position. Consequently, Ia leads V.

From the above discussion, it is concluded that if the synchronous motor is under-excited, it has a lagging power factor. As the excitation is increased, the power factor improves till it becomes unity at normal excitation. Under such conditions, the current drawn from the supply is minimum. If the excitation is further increased (i.e., over excitation), the motor power factor becomes leading. Note. The armature current (Ia) is minimum at unity p.f and increases as the power factor becomes poor, either leading or lagging.

2.16 Synchronous Condenser

A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor. An over-excited synchronous motor running on no-load in known as synchronous condenser. When such a machine is connected in parallel with induction motors or other devices that operate at low lagging power factor, the leading kVAR supplied by the synchronous condenser partly
neutralizes the lagging reactive kVAR of the loads. Consequently, the power factor of the system is improved. Fig: 2.29 shows the power factor improvement by synchronous condenser method. The 3 - f load takes current IL at low lagging power factor \( \cos \phi_L \). The synchronous condenser takes a current Im which leads the voltage by an angle \( \phi_m \). The resultant current I is the vector sum of Im and IL and lags behind the voltage by an angle \( \phi \). It is clear that \( \phi \) is less than \( \phi_L \) so that \( \cos \phi \) is greater than \( \cos \phi_L \). Thus the power factor is increased from \( \cos \phi_L \) to \( \cos \phi \). Synchronous condensers are generally used at major bulk supply substations for power factor improvement.

Fig: 2.29

**Advantages**

(i) By varying the field excitation, the magnitude of current drawn by the motor can be changed by any amount. This helps in achieving step less control of power factor.

(ii) The motor windings have high thermal stability to short circuit currents.

(iii) The faults can be removed easily.

**Disadvantages**

(i) There are considerable losses in the motor.

(ii) The maintenance cost is high.

(iii) It produces noise.

(iv) Except in sizes above 500 RVA, the cost is greater than that of static capacitors of the same rating.

(v) As a synchronous motor has no self-starting torque, therefore, an auxiliary equipment has to be provided for this purpose.

**2.17 Methods of starting synchronous motor**

There are three chief methods that are used to start a synchronous motor:

1. To reduce the speed of the rotating magnetic field of the stator to a low enough value that the rotor can easily accelerate and lock in with it during one half-cycle of the rotating magnetic
field’s rotation. This is done by reducing the frequency of the applied electric power. This method is usually followed in the case of inverter-fed synchronous motor operating under variable speed drive applications.

2. To use an external prime mover to accelerate the rotor of synchronous motor near to its synchronous speed and then supply the rotor as well as stator. Of course care should be taken to ensure that the directions of rotation of the rotor as well as that of the rotating magnetic field of the stator are the same. This method is usually followed in the laboratory- the synchronous machine is started as a generator and is then connected to the supply mains by following the synchronization or paralleling procedure. Then the power supply to the prime mover is disconnected so that the synchronous machine will continue to operate as a motor.

3. To use damper windings if these are provided in the machine. The damper windings are provided in most of the large synchronous motors in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load.

**Motor Starting by reducing the supply Frequency**

If the rotating magnetic field of the stator in a synchronous motor rotates at a low enough speed, there will be no problem for the rotor to accelerate and to lock in with the stator’s magnetic field. The speed of the stator magnetic field can then be increased to its rated operating speed by gradually increasing the supply frequency ‘\( f \)’ up to its normal 50- or 60-Hz value.

But the usual power supply systems generally regulate the frequency to be 50 or 60 Hz as the case may be. However, variable-frequency voltage source can be obtained from a dedicated generator only in the olden days and such a situation was obviously impractical except for very unusual or special drive applications. But the present day solid state power converters offer an easy solution to this. We now have the rectifier- inverter and cycloconverters, which can be used to convert a constant frequency AC supply to a variable frequency AC supply. With the development of such modern solid-state variable-frequency drive packages, it is thus possible to continuously control the frequency of the supply connected to the synchronous motor all the way from a fraction of a hertz up to and even above the normal rated frequency. If such a variable-frequency drive unit is included in a motor-control circuit to achieve speed control, then starting the synchronous motor is very easy—simply adjust the frequency to a very low value for starting, and then raise it up to the desired operating frequency for normal running.

When a synchronous motor is operated at a speed lower than the rated speed, its internal generated voltage (usually called the counter EMF) \( E_A = K \varphi \omega \) will be smaller than normal. As such the terminal voltage applied to the motor must be reduced proportionally with the frequency in order to keep the stator current within the rated value. Generally, the voltage in any variable-frequency power supply varies roughly linearly with the output frequency.

**Motor Starting with an External Motor**

The second method of starting a synchronous motor is to attach an external starting motor (pony motor) to it and bring the synchronous machine to near about its rated speed (but not exactly equal to it, as the synchronization process may fail to indicate the point of closure of the main switch connecting the synchronous machine to the supply system) with the pony motor. Then the output of the synchronous machine can be synchronised or paralleled with its power supply system as a
generator, and the pony motor can be detached from the shaft of the machine or the supply to the pony motor can be disconnected. Once the pony motor is turned OFF, the shaft of the machine slows down, the speed of the rotor magnetic field $B_R$ falls behind $B_{net}$, momentarily and the synchronous machine continues to operate as a motor. As soon as it begins to operate as a motor the synchronous motor can be loaded in the usual manner just like any motor.

This whole procedure is not as cumbersome as it sounds, since many synchronous motors are parts of motor-generator sets, and the synchronous machine in the motor-generator set may be started with the other machine serving as the starting motor. Moreover, the starting motor is required to overcome only the mechanical inertia of the synchronous machine without any mechanical load (load is attached only after the synchronous machine is paralleled to the power supply system). Since only the motor’s inertia must be overcome, the starting motor can have a much smaller rating than the synchronous motor it is going to start.

Generally most of the large synchronous motors have brushless excitation systems mounted on their shafts. It is then possible to use these exciters as the starting motors. For many medium-size to large synchronous motors, an external starting motor or starting by using the exciter may be the only possible solution, because the power systems they are tied to may not be able to handle the starting currents needed to use the damper (amortisseur) winding.

**Motor Starting by using damper (Amortisseur) Winding**

As already mentioned earlier most of the large synchronous motors are provided with damper windings, in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load. Damper windings are special bars laid into slots cut in the pole face of a synchronous machine and then shorted out on each end by a large shorting ring, similar to the squirrel cage rotor bars. A salient pole rotor with sets of damper windings is shown in Fig: 2.30 below.

![Fig: 2.30](image)

When the stator of such a synchronous machine is connected to the 3-Phase AC supply, the machine starts as a 3-Phase induction machine due to the presence of the damper bars, just like a squirrel cage
induction motor. Just as in the case of a 3-Phase squirrel cage induction motor, the applied voltage must be suitably reduced so as to limit the starting current to the safe rated value. Once the motor picks up to a speed near about its synchronous speed, the DC supply to its field winding is connected and the synchronous motor pulls into step i.e. it continues to operate as a Synchronous motor running at its synchronous speed.

2.18 Performance Characteristic

The effects of changes in mechanical or shaft load on armature current, power angle, and power factor can be seen from the phasor diagram shown in Fig: 2.31; As the applied stator voltage, frequency, and field excitation are assumed, constant. The initial load conditions, are represented by the thick lines. The effect of increasing the shaft load to twice its initial value are represented by the light lines indicating the new steady state conditions. When the shaft load is doubled both $I_a \cos \phi_i$ and $E_f \sin \delta$ are doubled. While redrawing the phasor diagrams to show new steady-state conditions, the line of action of the new $jI_a X_s$ phasor must be perpendicular to the new $I_a$ phasor. Furthermore, as shown in Fig: 2.31, if the excitation is not changed, increasing the shaft load causes the locus of the $E_f$ phasor to follow a circular arc, thereby increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in $\phi_i$; resulting in an increase in power factor.

As additional load is placed on the machine, the rotor continues to increase its angle of lag relative to the rotating magnetic field, thereby increasing both the angle of lag of the counter EMF phasor and the magnitude of the stator current. It is interesting to note that during all this load variation, however, except for the duration of transient conditions whereby the rotor assumes a new position in relation to the rotating magnetic field, the average speed of the machine does not change. As the load is being increased, a final point is reached at which a further increase in $\delta$ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. In fact as stated earlier, the rotor poles at this point, will fall behind the stator poles such that they now come under the influence of like poles and the force of attraction no longer exists. Thus, the point of maximum torque occurs at a power angle of approximately $90^\circ$ for a cylindrical-rotor machine. This maximum value of torque that causes a synchronous motor to pull out of synchronism is called the pull-out torque. In actual practice, the motor will never be operated at power angles close to $90^\circ$ as armature current will be many times its rated value at this load.
**Effect of changes in field excitation on synchronous motor performance**

As increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating magnetic poles of the stator. This will obviously result in a smaller power angle. When the shaft load is assumed to be constant, the steady-state value of $E_f \sin \delta$ must also be constant. An increase in $E_f$ will cause a transient increase in $E_f \sin \delta$, and the rotor will accelerate. As the rotor changes its angular position, $\delta$ decreases until $E_f \sin \delta$ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed, as it should run only at the synchronous speed. This change in angular position of the rotor magnets relative to the poles of rotating magnetic field of the stator occurs in a fraction of a second. The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a constant shaft load, from a constant voltage, constant frequency supply, is illustrated in Fig: 2.32. For a constant shaft load,

$$E_{f1} \sin \delta_1 = E_{f2} \sin \delta_2 = E_{f3} \sin \delta_3 = E_f \sin \delta$$

This is shown in Fig. 57, where the locus of the tip of the $E_f$ phasor is a straight line parallel to the $V_T$ phasor. Similarly, for a constant shaft load,

$$I_{a1} \cos \phi_{i1} = I_{a2} \cos \phi_{i2} = I_{a3} \cos \phi_{i3} = I_a \cos \phi_i$$

This is also shown in Fig. 57, where the locus of the tip of the $I_a$ phasor is a line perpendicular to the $V_T$ phasor.
Note that increasing the excitation from $E_f1$ to $E_f3$ in Fig: 2.32 caused the phase angle of the current phasor with respect to the terminal voltage $V_T$ (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called normal excitation. Excitation greater than normal is called over excitation, and excitation less than normal is called under excitation. Furthermore, as indicated in Fig: 2.32, when operating in the overexcited mode, $|E_f| > |V_T|$. In fact a synchronous motor operating under over excitation condition is sometimes called a synchronous condenser.

**Power Factor Characteristic of Synchronous Motors**

In an induction motor, only one winding (i.e., stator winding) produces the necessary flux in the machine. The stator winding must draw reactive power from the supply to set up the flux. Consequently, induction motor must operate at lagging power factor. But in a synchronous motor, there are two possible sources of excitation; alternating current in the stator or direct current in the rotor. The required flux may be produced either by stator or rotor or both.

(i) If the rotor exciting current is of such magnitude that it produces all the required flux, then no magnetizing current or reactive power is needed in the stator. As a result, the motor will operate at unity power factor.

(ii) If the rotor exciting current is less (i.e., motor is under-excited), the deficit in flux is made up by the stator. Consequently, the motor draws reactive power to provide for the remaining flux. Hence motor will operate at a lagging power factor.

(iii) If the rotor exciting current is greater (i.e., motor is over-excited), the excess flux must be counterbalanced in the stator. Now the stator, instead of absorbing reactive power, actually delivers reactive power to the 3-phase line. The motor then behaves like a source of reactive power, as if it
were a capacitor. In other words, the motor operates at a leading power factor.
To sum up, a synchronous motor absorbs reactive power when it is under excited and delivers reactive power to source when it is over-excited.

2.19 Hunting and Damper Winding:

**Hunting:**

Sudden changes of load on synchronous motors may sometimes set up oscillations that are superimposed upon the normal rotation, resulting in periodic variations of a very low frequency in speed. This effect is known as hunting or phase-swinging. Occasionally, the trouble is aggravated by the motor having a natural period of oscillation approximately equal to the hunting period. When the synchronous motor phase-swinging into the unstable region, the motor may fall out of synchronism.

**Damper winding:**

The tendency of hunting can be minimized by the use of a damper winding. Damper windings are placed in the pole faces. No emfs are induced in the damper bars and no current flows in the damper winding, which is not operative. Whenever any irregularity takes place in the speed of rotation, however, the polar flux moves from side to side of the pole, this movement causing the flux to move backwards and forwards across the damper bars. Emfs are induced in the damper bars forwards across the damper winding. These tend to damp out the superimposed oscillatory motion by absorbing its energy. The damper winding, thus, has no effect upon the normal average speed, it merely tends to damp out the oscillations in the speed, acting as a kind of electrical flywheel. In the case of a three-phase synchronous motor the stator currents set up a rotating mmf rotating at uniform speed and if the rotor is rotating at uniform speed, no emfs are induced in the damper bars. Fig: 2.33 shows a salient pole synchronous motor with damper winding.
2.20 Synchronous Induction Motor

In the applications where high starting torque and constant speed are desired then synchronous induction motor can be used. It has the advantages of both synchronous motor and induction motor. The synchronous motor gives constant speed whereas induction motors can be started against full load torque.

Consider a normal slip ring induction motor having three phase winding on the rotor. The motor is connected to the exciter which gives D.C. supply to the rotor through slip rings. One phase carries full D.C. current while the other two carries half the full D.C. current as they are connected in parallel. Due to this D.C. excitation, permanent poles (N and S) formed on the rotor.

Initially it is run as a slip ring induction motor with the help of starting resistances. When the resistances are cut out the motor runs with a slip. Now the connections are changed and the exciter is connected in series with the rotor windings which will remain in the circuit permanently.

As the motor is running as induction motor initially high starting torque (up to twice full load value) can be developed. When the D.C. excitation is provided it is pulled into synchronism and starts running at constant speed. Thus synchronous induction motor provides constant speed, large starting torque, low starting current and power factor correction.
Acknowledgement

The committee members gratefully acknowledge google, scribd, NPTEL, openoffice, sumatra pdf, scilab for myriad suggestions and help for preparing this lecture note. The committee members also wants to express their gratitude to the persons out there who think knowledge should be free and be accessible and sharable without any restrictions so that every single person on this planet has the same opportunity to explore, expand and become enlightened by the collective gifts of humankind.

However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books and above all confer with the concern faculty for thorough knowledge of this authoritative subject of electrical engineering.

Text / Reference Books

1. Electrical Machinery [7th Ed.] by P. S. Bimbhra
   Publisher- Khanna Publisher
2. Generalized Theory of Electrical Machines [2nd Ed.] by P.S. Bimbhra
   Publisher- Khanna Publisher
3. The Performance and Design of Alternating Current Machines [3rd Ed.] by M. G. Say
   Publisher- CBS Publisher
   Publisher- McGraw Hill Education (India) Private Limited
   Publisher- McGraw Hill Education (India) Private Limited
   Publisher- McGraw Hill Education (India) Private Limited
   Publisher- John Wiley & Sons
8. Electric Machinery and Transformers [2nd Ed.] by Irving Kosow
   Publisher- Pearson India
   Publisher- McGraw Hill Education (India) Private Limited
10. A Course in Electrical Engineering Vol.-I & Vol.-II by Chester L. Dawes, S. B.
    Publisher- McGraw Hill Book Company Inc.
    Publisher- McGraw Hill Book Company Inc.
    Publisher- Dhanpat Rai Publications
    Publisher- McGraw Hill Education (India) Private Limited
    Publisher- CBS Publisher

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LECTURE NOTES ON

ELECTRICAL MACHINE-II

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[Part-II]
[Module-III & IV]

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\textit{(4th SEMESTER)}

ELECTRICAL MACHINES-II

Subject Code: BEE 1401

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SYLLABUS/ TOPICS COVERED

Three Phase Induction Motors: Types, Construction and principle of operation, 3 phase Induction Motor, general phasor diagram, equivalent circuit, power and torque relations, condition for maximum torque, circle diagram, Performance characteristics, effect of rotor resistance on speed torque characteristics, stable & unstable region of operation, Operation with unbalanced supply voltage. Starting: Starting of 3 phase induction motors, high starting torque motors, speed control, rheostatic method, pole changing method cascade control of speed, Double cage induction motor, Cogging and Crawling of Induction motor, Induction generator

[Topics are arranged as per above sequence]
Module-III

3.1 Three Phase Induction Motor

The most common type of AC motor being used throughout the work today is the "Induction Motor". Applications of three-phase induction motors of size varying from half a kilowatt to thousands of kilowatts are numerous. They are found everywhere from a small workshop to a large manufacturing industry.

The advantages of three-phase AC induction motor are listed below:

- Simple design
- Rugged construction
- Reliable operation
- Low initial cost
- Easy operation and simple maintenance
- Simple control gear for starting and speed control
- High efficiency.

Induction motor is originated in the year 1891 with crude construction (The induction machine principle was invented by NIKOLA TESLA in 1888.). Then an improved construction with distributed stator windings and a cage rotor was built.

The slip ring rotor was developed after a decade or so. Since then a lot of improvement has taken place on the design of these two types of induction motors. Lot of research work has been carried out to improve its power factor and to achieve suitable methods of speed control.

3.2 Types and Construction of Three Phase Induction Motor

Three phase induction motors are constructed into two major types:

1. Squirrel cage Induction Motors
2. Slip ring Induction Motors

3.2.1 Squirrel cage Induction Motors

(a) Stator Construction

The induction motor stator resembles the stator of a revolving field, three phase alternator. The stator or the stationary part consists of three phase winding held in place in the slots of a laminated steel core which is enclosed and supported by a cast iron or a steel frame as shown in Fig: 3.1(a).
The phase windings are placed 120 electrical degrees apart and may be connected in either star or delta externally, for which six leads are brought out to a terminal box mounted on the frame of the motor. When the stator is energized from a three phase voltage it will produce a rotating magnetic field in the stator core.

(b) Rotor Construction

The rotor of the squirrel cage motor shown in Fig: 3.1(b) contains no windings. Instead it is a cylindrical core constructed of steel laminations with conductor bars mounted parallel to the shaft and embedded near the surface of the rotor core.

These conductor bars are short circuited by an end rings at both end of the rotor core. In large machines, these conductor bars and the end rings are made up of copper with the bars brazed or welded to the end rings shown in Fig: 3.1(b). In small machines the conductor bars and end rings are sometimes made of aluminium with the bars and rings cast in as part of the rotor core. Actually the entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

The rotor or rotating part is not connected electrically to the power supply but has voltage induced in it by transformer action from the stator. For this reason, the stator is sometimes called the primary and the rotor is referred to as the secondary of the motor since the motor operates on the principle of induction and as the construction of the rotor with the bars and end rings resembles a squirrel cage, the squirrel cage induction motor is used.

The rotor bars are not insulated from the rotor core because they are made of metals having less resistance than the core. The induced current will flow mainly in them. Also the rotor bars are usually not quite parallel to the rotor shaft but are mounted in a slightly skewed position. This feature tends to produce a more uniform rotor field and torque. Also it helps to reduce some of the internal magnetic noise when the motor is running.
(c) End Shields

The function of the two end shields is to support the rotor shaft. They are fitted with bearings and attached to the stator frame with the help of studs or bolts attention.

3.2.2 Slip ring Induction Motors

(a) Stator Construction

The construction of the slip ring induction motor is exactly similar to the construction of squirrel cage induction motor. There is no difference between squirrel cage and slip ring motors.

(b) Rotor Construction

The rotor of the slip ring induction motor is also cylindrical or constructed of lamination.

Squirrel cage motors have a rotor with short circuited bars whereas slip ring motors have wound rotors having "three windings" each connected in star.

The winding is made of copper wire. The terminals of the rotor windings of the slip ring motors are brought out through slip rings which are in contact with stationary brushes as shown in Fig: 3.2.

THE ADVANTAGES OF THE SLIPRING MOTOR ARE

- It has susceptibility to speed control by regulating rotor resistance.
- High starting torque of 200 to 250% of full load value.
- Low starting current of the order of 250 to 350% of the full load current.

Hence slip ring motors are used where one or more of the above requirements are to be met.
### 3.2.3 Comparison of Squirrel Cage and Slip Ring Motor

<table>
<thead>
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<th>Sl.No.</th>
<th>Property</th>
<th>Squirrel cage motor</th>
<th>Slip ring motor</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td><strong>Rotor Construction</strong></td>
<td>Bars are used in rotor. Squirrel cage motor is very simple, rugged and long lasting. No slip rings and brushes</td>
<td>Winding wire is to be used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wound rotor required attention.</td>
<td>Slip ring and brushes are needed also need frequent maintenance.</td>
</tr>
<tr>
<td>2.</td>
<td><strong>Starting</strong></td>
<td>Can be started by D.O.L., star-delta, auto transformer starters</td>
<td>Rotor resistance starter is required.</td>
</tr>
<tr>
<td>3.</td>
<td><strong>Starting torque</strong></td>
<td>Low</td>
<td>Very high</td>
</tr>
<tr>
<td>4.</td>
<td><strong>Starting Current</strong></td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>5.</td>
<td><strong>Speed variation</strong></td>
<td>Not easy, but could be varied in large steps by pole changing or through smaller incremental steps through thyristors or by frequency variation.</td>
<td>Easy to vary speed. Speed change is possible by inserting rotor resistance using thyristors or by using frequency variation injecting emf in the rotor circuit cascading.</td>
</tr>
<tr>
<td>6.</td>
<td><strong>Maintenance</strong></td>
<td>Almost maintenance ZERO maintenance</td>
<td>Requires frequent maintenance</td>
</tr>
<tr>
<td>7.</td>
<td><strong>Cost</strong></td>
<td>Low</td>
<td>High</td>
</tr>
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### 3.3 Principle of Operation

The operation of a 3-phase induction motor is based upon the application of Faraday Law and the Lorentz force on a conductor. The behaviour can readily be understood by means of the following example.

Consider a series of conductors of length \( l \), whose extremities are short-circuited by two bars A and B (Fig.3.3 a). A permanent magnet placed above this conducting ladder, moves rapidly to the right at a speed \( v \), so that its magnetic field \( B \) sweeps across the conductors. The following sequence of events then takes place:
1. A voltage $E = Blv$ is induced in each conductor while it is being cut by the flux (Faraday law).

2. The induced voltage immediately produces a current $I$, which flows down the conductor underneath the pole face, through the end-bars, and back through the other conductors.

3. Because the current-carrying conductor lies in the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force).

4. The force always acts in a direction to drag the conductor along with the magnetic field. If the conducting ladder is free to move, it will accelerate toward the right. However, as it picks up speed, the conductors will be cut less rapidly by the moving magnet, with the result that the induced voltage $E$ and the current $I$ will diminish. Consequently, the force acting on the conductors will also decrease. If the ladder were to move at the same speed as the magnetic field, the induced voltage $E$, the current $I$, and the force dragging the ladder along would all become zero.

Fig: 3.3

In an induction motor the ladder is closed upon itself to form a squirrel-cage (Fig.3.3b) and the moving magnet is replaced by a rotating field. The field is produced by the 3-phase currents that flow in the stator windings.

3.4 Rotating Magnetic Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns (Fig.3.4). Coils that are diametrically opposite are connected in series by means of three jumpers
that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, which are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magneto motive forces that act in the same direction.

The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.

For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the centre of the magnetic poles. The result is illustrated in Fig.3.5. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.

Fig: 3.4 Elementary stator having terminals A, B, C connected to a 3-phase source (not shown). Currents flowing from line to neutral are considered to be positive.

Fig: 3.5 Air gap flux density distribution.
The flux density distribution in the air gap can be expressed as:

$$B(\theta) = B_{\max} \cos \theta$$

The air gap flux per pole, $\phi_p$, is:

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) l r d\theta = 2B_{\max} lr$$

Where,

- $l$ is the axial length of the stator.
- $r$ is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of $N$ turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig.3.5). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil $aa'$ will be maximum.

$$= N\phi_p \text{ at } \omega t = 0^\circ \text{ (Fig.3.5a) and zero at } \omega t = 90^\circ.$$ The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle $\omega t$.

Hence,

$$\lambda_a(\omega t) = N\phi_p \cos \omega t$$

Therefore, the voltage induced in phase coil $aa'$ is obtained from Faraday law as:

$$e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\max} \sin \omega t$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{\max} \sin(\omega t - 120)$$

$$e_c = E_{\max} \sin(\omega t + 120).$$

The $rms$ value of the induced voltage is:

$$E_{rms} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44fN\phi_p$$

Where $f$ is the frequency in hertz. Above equation has the same form as that for the induced voltage in transformers. However, $\phi_P$ represents the flux per pole of the machine.
The above equation also shows the rms voltage per phase. The N is the total number of series turns per phase with the turns forming a concentrated full-pitch winding. In an actual AC machine each phase winding is distributed in a number of slots for better use of the iron and copper and to improve the waveform. For such a distributed winding, the EMF induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the EMF is less than their numerical sum when they are connected in series for the phase winding. A reduction factor $K_W$, called the winding factor, must therefore be applied. For most three-phase machine windings $K_W$ is about 0.85 to 0.95.

Therefore, for a distributed phase winding, the rms voltage per phase is

$$E_{rms} = 4.44f N_{ph} \phi_p K_W$$

Where $N_{ph}$ is the number of turns in series per phase.

### 3.5 Alternate Analysis for Rotating Magnetic Field

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to $1.5 \phi_m$ where $m$ is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in Fig. 3.6 (i). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as $I_x$, $I_y$ and $I_z$ [See Fig. 3.6 (ii)]. Referring to Fig. 3.6 (ii), the fluxes produced by these currents are given by:

$$\phi_X = \phi_m \sin \omega t$$
$$\phi_Y = \phi_m \sin (\omega t - 120^\circ)$$
$$\phi_Z = \phi_m \sin (\omega t - 240^\circ)$$

Here $\phi_m$ is the maximum flux due to any phase. Above figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to $1.5 \phi_m$.

At instant 1 [See Fig. 3.6 (ii) and Fig. 3.6 (iii)], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward
in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5 $\phi_m$ as proved under:

At instant 1, $\omega t = 0^\circ$. Therefore, the three fluxes are given by:

$$\phi_x = 0; \quad \phi_y = \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of $-\phi_y$ and $\phi_z$ is the resultant flux $\phi_r$

So,

Resultant flux, $\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos 60^\circ = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5 \phi_m$

At instant 2 [Fig: 3.7 (ii)], the current is maximum (negative) in $\phi_y$ phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5 $\phi_m$ as proved under:

At instant 2, $\omega t = 30^\circ$. Therefore, the three fluxes are given by:

$$\phi_x = \phi_m \sin 30^\circ = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90^\circ) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210^\circ) = \frac{\phi_m}{2}$$

The phasor sum of $\phi_x$, $-\phi_y$ and $\phi_z$ is the resultant flux $\phi_r$

Phasor sum of $\phi_x$ and $\phi_z$, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos 120^\circ = \frac{\phi_m}{2}$

Phasor sum of $\phi'_r$ and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$

Note that resultant flux is displaced $30^\circ$ clockwise from position 1.
At instant 3[Fig: 3.7 (iii)], current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y are $0.866 \times$ max. value). The magnitude of resultant flux is $1.5 \phi_m$ as proved under:
At instant 3, \( \theta = 60^\circ \). Therefore, the three fluxes are given by:

\[
\begin{align*}
\phi_x &= \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2} \phi_m; \\
\phi_y &= \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2} \phi_m; \\
\phi_z &= \phi_m \sin(-180^\circ) = 0
\end{align*}
\]

The resultant flux \( \phi_r \) is the phasor sum of \( \phi_x \) and \( -\phi_y \) (\( \therefore \phi_z = 0 \)).

\[
\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 1.5 \phi_m
\]

Note that resultant flux is displaced 60\(^\circ\) clockwise from position 1.

At instant 4 [Fig: 3.7 (iv)], the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are \(0.5 \times \text{max. value}\)). This establishes a resultant flux downward as shown under:
At instant 4, \(\omega t = 90^\circ\). Therefore, the three fluxes are given by:

\[
\phi_x = \phi_m \sin 90^\circ = \phi_m
\]

\[
\phi_y = \phi_m \sin (-30^\circ) = -\frac{\phi_m}{2}
\]

\[
\phi_z = \phi_m \sin (-150^\circ) = -\frac{\phi_m}{2}
\]

The phasor sum of \(\phi_x\), \(-\phi_y\) and \(-\phi_z\) is the resultant flux \(\phi_r\).

Phasor sum of \(-\phi_z\) and \(-\phi_y\), \(\phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}\)

Phasor sum of \(\phi'_r\) and \(\phi_x\), \(\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m\)

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= \(1.5 \phi_m\), where \(\phi_m\) is the maximum flux due to any phase).

### 3.5.1 Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed \((N_s)\). Referring to Fig. 3.6 (ii), the time instant 4 represents the completion of one-quarter cycle of alternating current \(I_x\) from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 [Fig. 3.6 (ii)] or one complete cycle of current \(I_x\) from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, for \(P\) poles, the rotating field makes one revolution in \(P/2\) cycles of current.

\[\therefore \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}\]

or \(\text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}\)

Since revolutions per second is equal to the revolutions per minute \((N_s)\) divided by 60 and the number of cycles per second is the frequency \(f\),

\[\therefore f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}\]

or \(N_s = \frac{120 f}{P}\)

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.
3.5.2 Direction of rotating magnetic field

The phase sequence of the three-phase voltage applied to the stator winding in Fig. 3.6 (ii) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counter clockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

3.5.3 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the stator field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

\[
% \text{ slip, } s = \frac{N_s - N}{N_s} \times 100
\]

(i) The quantity \(N_s - N\) is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., \(N = 0\)), slip, \(s = 1\) or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

3.5.4 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;
Frequency = \frac{NP}{120}

where \( N \) = Relative speed between magnetic field and the winding
\( P \) = Number of poles

For a rotor speed \( N \), the relative speed between the rotating flux and the rotor is \( N_s - N \). Consequently, the rotor current frequency \( f' \) is given by:

\[
f' = \frac{(N_s - N)P}{120} = s \frac{N_s P}{120} = sf' \quad \text{where} \quad s = \frac{N_s - N}{N_s}
\]

i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., \( s = 1 \)), the frequency of rotor current is the same as that of supply frequency (\( f' = sf' = 1 \times f = f \)).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip \( s \) and hence rotor current frequency decreases.

### 3.6 Phasor Diagram of Three Phase Induction Motor

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor finding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. 3.8 shows the equivalent circuit per phase for an induction motor. Let discuss the stator and rotor circuits separately.
**Stator circuit.** In the stator, the events are very similar to those in the transformer primary. The applied voltage per phase to the stator is $V_1$ and $R_1$ and $X_1$ are the stator resistance and leakage reactance per phase respectively. The applied voltage $V_1$ produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f. $E_1$ is induced in the stator winding and mutually induced e.m.f. $E_2' = s E_2 = s K E_1$ where $K$ is transformation ratio) is induced in the rotor winding. The flow of stator current $I_1$ causes voltage drops in $R_1$ and $X_1$.

\[
V_1 = -E_1 + I_1 (R_1 + j X_1) \quad \text{...phasor sum}
\]

When the motor is at no-load, the stator winding draws a current $I_0$. It has two components viz., (i) which supplies the no-load motor losses and (ii) magnetizing component $I_m$ which sets up magnetic flux in the core and the air gap. The parallel combination of $R_c$ and $X_m$, therefore, represents the no-load motor losses and the production of magnetic flux respectively.

\[
I_0 = I_w + I_m
\]

**Rotor circuit.** Here $R_2$ and $X_2$ represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip $s$, the rotor reactance will be $X_2$. The induced voltage/phase in the rotor is $E_2' = s E_2 = s K E_1$. Since the rotor winding is short-circuited, the whole of e.m.f. $E_2'$ is used up in circulating the rotor current $I_2'$.

\[
E_2' = I_2' (R_2 + j s X_2)
\]

The rotor current $I_2'$ is reflected as $I_2'' (= K I_2')$ in the stator. The phasor sum of $I_2''$ and $I_0$ gives the stator current $I_1$.

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.
It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed $N_s$. The stator currents produce a magnetic flux which rotates at a speed $N_s$. At slip $s$, the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$\frac{120 f'}{P} = \frac{120 s f}{P} = s N_s$$

But the rotor is revolving at a speed of $N$ relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= sN_s + N = (N_s - N) + N = N_s$$

Thus no matter what the value of slip $s$, the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings. Fig. 3.9 shows the phasor diagram of induction motor.
3.7 Equivalent Circuit of Three Phase Induction Motor

Fig. 3.10 (i) shows the equivalent circuit per phase of the rotor at slip s. The rotor phase current is given by:

\[ I'_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \]

Mathematically, this value is unaltered by writing it as:

\[ I'_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}} \]

As shown in Fig. 3.10 (ii), we now have a rotor circuit that has a fixed reactance \( X_2 \) connected in series with a variable resistance \( R_2/s \) and supplied with constant voltage \( E_2 \). Note that Fig. 3.10 (ii) transfers the variable to the resistance without altering power or power factor conditions.

![Equivalent Circuit Diagrams](image)

Fig: 3.10

The quantity \( R_2/s \) is greater than \( R_2 \) since \( s \) is a fraction. Therefore, \( R_2/s \) can be divided into a fixed part \( R_2 \) and a variable part \( (R_2/s - R_2) \) i.e.,

\[ \frac{R_2}{s} = R_2 + R_2 \left( \frac{1}{s} - 1 \right) \]
Fig. 3.10 (iii) shows the equivalent rotor circuit along with load resistance $R_L$.

Now Fig. 3.11 shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance $R_L$ given by;

\[
R_L = R_2 \left( \frac{1}{s} - 1 \right)
\]

Fig. 3.10 (iii) shows the equivalent rotor circuit along with load resistance $R_L$.

Now Fig. 3.11 shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance $R_L$ given by;

\[
R_L = R_2 \left( \frac{1}{s} - 1 \right)
\]

----- (i)

The circuit shown in Fig. 3.11 is similar to the equivalent circuit of a transformer with secondary load equal to $R_2$ given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio $K (= E_2/E_1)$.

Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load $R_L$ given by eq. (i). The power delivered to $R_L$ represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. 3.11 is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio $K$. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by $K^2$ whereas current should be multiplied by $K$. The equivalent circuit of an induction motor referred to primary is shown in Fig. 3.12.
Note that the element (i.e., $R'L$) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

(i) At no-load, the slip is practically zero and the load $R'L$ is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.

(ii) At standstill, the slip is unity and the load $R'L$ is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.

(iii) When the motor is running under load, the value of $R'L$ will depend upon the value of the slip $s$. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.

(iv) The equivalent electrical resistance $R'L$ related to mechanical load is slip or speed dependent. If the slip $s$ increases, the load $R'L$ decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor shaft.

3.8 Power and Torque Relations of Three Phase Induction Motor

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. 3.13 shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).
This is quite apparent from the equivalent circuit shown in Fig: 3.13.

\[ (i) \quad \text{Total electrical load} = R'_2 \left( \frac{1}{s} - 1 \right) + R'_2 = \frac{R'_2}{s} \]

Power input to stator = \( 3V_1 I_1 \cos \phi_1 \)

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

\[ (ii) \quad \text{Rotor input} = \frac{3(I''_2)^2 R'_2}{s} \]

Rotor Cu loss = \( 3(I''_2)^2 R'_2 \)

Total mechanical power developed by the rotor is

\[ P_m = \text{Rotor input} - \text{Rotor Cu loss} \]
\[ = \frac{3(I''_2)^2 R'_2}{s} \left( 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2 \left( \frac{1}{s} - 1 \right) \right) \]

This is quite apparent from the equivalent circuit shown in Fig: 3.13.
(iii) If $T_g$ is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N T_g}{60}$$

or

$$3(I''_2)^2 R'_2 \left( \frac{1}{s} - 1 \right) = \frac{2\pi N T_g}{60}$$

or

$$3(I''_2)^2 R'_2 \left( \frac{1 - s}{s} \right) = \frac{2\pi N s (1 - s)}{60} T_g$$

or

$$3(I''_2)^2 R'_2 \left( \frac{1 - s}{s} \right) = \frac{2\pi N_s (1 - s)}{60} T_g \quad \text{[:: N = N_s (1 - s)]}$$

\[\therefore \quad T_g = \frac{3(I''_2)^2 R'_2 / s}{2\pi N_s / 60} \quad N \cdot m\]

or

$$T_g = 9.55 \frac{3(I''_2)^2 R'_2 / s}{N_s} \quad N \cdot m$$

Note that shaft torque $T_{sh}$ will be less than $T_g$ by the torque required to meet windage and frictional losses.

### 3.9 Induction Motor Torque

The mechanical power $P$ available from any electric motor can be expressed as:

$$P = \frac{2\pi NT}{60} \quad \text{watts}$$

where $N =$ speed of the motor in r.p.m.

$T =$ torque developed in N-m

\[\therefore \quad T = \frac{60}{2\pi N} \frac{P}{N} = 9.55 \frac{P}{N} \quad N \cdot m\]

If the gross output of the rotor of an induction motor is $P_m$ and its speed is $N$ r.p.m., then gross torque $T$ developed is given by:

$$T_g = 9.55 \frac{P_m}{N} \quad N \cdot m$$

Similarly, $T_{sh} = 9.55 \frac{P_{out}}{N} \quad N \cdot m$

**Note.** Since windage and friction loss is small, $T_g = T_{sh}$. This assumption hardly leads to any significant error.
3.10 Rotor Output

If $T_g \text{ newton-metre}$ is the gross torque developed and $N \text{ r.p.m.}$ is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi NT_g}{60} \text{ watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed $N_s$.

$$\therefore \text{Rotor input} = \frac{2\pi N_s T_g}{60} \text{ watts}$$

$$\therefore \text{Rotor Cu loss} = \text{Rotor input} - \text{Rotor output} = \frac{2\pi T_g}{60} (N_s - N)$$

(i) \[ \frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s \]

$$\therefore \text{Rotor Cu loss} = s \times \text{Rotor input}$$

(ii) \quad \text{Gross rotor output, } P_m = \text{Rotor input} - \text{Rotor Cu loss} = \text{Rotor input} - s \times \text{Rotor input}

$$\therefore P_m = \text{Rotor input} (1 - s)$$

(iii) \[ \frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s} \]

(iv) \[ \frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1 - s} \]

It is clear that if the input power to rotor is $“Pr”$ then $“s.Pr”$ is lost as rotor Cu loss and the remaining $(1 - s) Pr$ is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

**Note.**

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses are neglected, then,

$$\text{Gross rotor output} = \text{Useful output}$$

$$\text{Rotor input} = \text{Stator input}$$

$$\therefore \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is $1 - s$. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is $1 - 0.125 = 0.875$ or 87.5%.
3.11.1 Torque Equations

The gross torque $T_g$ developed by an induction motor is given by:

$$T_g = \frac{\text{Rotor input}}{2\pi N_s}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi N_s}$$

... $N_s$ is r.p.s.

Now,

$$\text{Rotor input} = \frac{\text{Rotor Cu loss}}{s} = \frac{3(I_2')^2 R_2}{s} \quad \text{(i)}$$

As shown in Sec. 8.16, under running conditions,

$$I_2' = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} = \frac{s K E_1}{\sqrt{R_2^2 + (s X_2)^2}}$$

where $K = \text{Transformation ratio} = \frac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \text{Rotor input} = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of $I_2'$ in eq.(i))

Also,

$$\therefore \text{Rotor input} = 3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of $I_2'$ in eq.(i))

$$\therefore T_g = \frac{\text{Rotor input}}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

...in terms of $E_2$

$$= \frac{3}{2\pi N_s} \times \frac{s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$$

...in terms of $E_1$

Note that in the above expressions of $T_g$, the values $E_1$, $E_2$, $R_2$ and $X_2$ represent the phase values.
3.11.2 Rotor Torque

The torque $T$ developed by the rotor is directly proportional to:

(i) rotor current  
(ii) rotor e.m.f.  
(iii) power factor of the rotor circuit

\[ T \propto E_2 I_2 \cos \phi_2 \]

or

\[ T = KE_2 I_2 \cos \phi_2 \]

where

$E_2 =$ rotor e.m.f. at standstill  
$I_2 =$ rotor current at standstill  
$\phi_2 =$ rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

3.11.3 Starting Torque ($T_s$)

Let,

$E_2 =$ rotor e.m.f. per phase at standstill  
$X_2 =$ rotor reactance per phase at standstill  
$R_2 =$ rotor resistance per phase

Rotor impedance/phase,  

\[ Z_2 = \sqrt{R_2^2 + X_2^2} \]

...at standstill

Rotor current/phase,  

\[ I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \]

...at standstill

Rotor p.f.,  

\[ \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \]

...at standstill

\[ T_s = KE_2 I_2 \cos \phi_2 \]

\[ = KE_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \]

\[ = \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \]

Generally, the stator supply voltage $V$ is constant so that flux per pole $\phi$ set up by the stator is also fixed. This in turn means that e.m.f. $E_2$ induced in the rotor will be constant.
3.11.4 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

It can be shown that \( K = \frac{3}{2} \pi N_s \).

\[
T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}
\]

Note that here \( N_s \) is in r.p.s.

### 3.11.4 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now

\[
T_s = \frac{K_1 R_2}{R_2^2 + X_2^2}
\]  \hspace{1cm} (i)

Differentiating eq. (i) w.r.t. \( R_2 \) and equating the result to zero, we get,

\[
\frac{dT_s}{dR_2} = K_1 \left[ \frac{1}{R_2^2 + X_2^2} - \frac{R_2 (2 R_2)}{(R_2^2 + X_2^2)^2} \right] = 0
\]

or

\[
R_2^2 + X_2^2 = 2 R_2^2
\]

or

\[
R_2 = X_2
\]

Hence starting torque will be maximum when:

\[
Rotor \text{ resistance/phase} = \text{Standstill rotor reactance/phase}
\]

Under the condition of maximum starting torque, \( \phi_2 = 45^\circ \) and rotor power factor is 0.707 lagging.
Fig. 3.14 shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when $R_2 = X_2$. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

### 3.11.5 Effect of Change of Supply Voltage

The starting torque can be expressed as:

$$T_s = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Since $E_2 \propto \text{Supply voltage } V$

$$\therefore T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where $K_2$ is another constant.

$$\therefore T_s \propto V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.
3.12 Circle Diagram

To analyse the three phase induction motor performance using circle diagram we need to determine the equivalent circuit parameters of the machine.

3.12.1 Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch (\(R_c - X_m\)) to the input terminals as shown in Fig. 3.15. This step has been taken on the assumption that voltage drop in \(R_1\) and \(X_1\) is small and the terminal voltage \(V_1\) does not appreciably differ from the induced voltage \(E_1\). Fig. 3.15 shows the approximate equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

![Fig: 3.15](image)

The above approximate circuit of induction motor is not so readily justified as with the transformer. This is due to the following reasons:

(i) Unlike that of a power transformer, the magnetic circuit of the induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the power transformer. Consequently, the exact equivalent circuit must be used for accurate results.

(ii) The relative values of \(X_1\) and \(X_2\) in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit.

(iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

In spite of the above drawbacks of approximate equivalent circuit, it yields results that are satisfactory for large motors. However, approximate equivalent circuit is not justified for small motors.
3.12.2 Tests to Determine the Equivalent Circuit Parameters

In order to find values for the various elements of the equivalent circuit, tests must be conducted on a particular machine, which is to be represented by the equivalent circuit. In order to do this, we note the following.

1. When the machine is run on no-load, there is very little torque developed by it. In an ideal case where there is no mechanical losses, there is no mechanical power developed at no-load. Recalling the explanations in the section on torque production, the flow of current in the rotor is indicative of the torque that is produced. If no torque is produced, one may conclude that no current would be flowing in the rotor either. The rotor branch acts like an open circuit. This conclusion may also be reached by reasoning that when there is no load, an ideal machine will run up to its synchronous speed where the slip is zero resulting in an infinite impedance in the rotor branch.

2. When the machine is prevented from rotation, and supply is given, the slip remains at unity. The elements representing the magnetizing branch $R_m$ & $X_m$ are high impedances much larger than $R_r$ & $X_r$, in series. Thus, in the exact equivalent circuit of the induction machine, the magnetizing branch may be neglected.

From these considerations, we may reduce the induction machine equivalent circuit of Fig.3.13 & Fig: 3.15 to those shown in Fig: 3.16.

![Fig: 3.16](image)

These two observations and the reduced equivalent circuits are used as the basis for the two most commonly used tests to find out the equivalent circuit parameters — the blocked rotor test and no load test. They are also referred to as the short circuit test and open circuit test respectively in conceptual analogy to the transformer.
1. No-load test

The behaviour of the machine may be judged from the equivalent circuit of Fig: 3.16 (a). The current drawn by the machine causes a stator-impedance drop and the balance voltage is applied across the magnetizing branch. However, since the magnetizing branch impedance is large, the current drawn is small and hence the stator impedance drop is small compared to the applied voltage (rated value). This drop and the power dissipated in the stator resistance are therefore neglected and the total power drawn is assumed to be consumed entirely as core loss. This can also be seen from the approximate equivalent circuit, the use of which is justified by the foregoing arguments. This test therefore enables us to compute the resistance and inductance of the magnetizing branch in the following manner.

Let applied voltage = $V_s$. Then current drawn is given by

$$I_s = \frac{V_s}{R_m} + \frac{V_s}{jX_m}$$

The power drawn is given by

$$P_s = \frac{V_s^2}{R_m} \Rightarrow R_m = \frac{V_s^2}{P_s}$$

$V_s$, $I_s$ and $P_s$ are measured with appropriate meters. With $R_m$ known by above equation, $X_m$ also can be found. The current drawn is at low power factor and hence a suitable wattmeter should be used.

2. Blocked-rotor Test

In this test the rotor is prevented from rotation by mechanical means and hence the name. Since there is no rotation, slip of operation is unity, $s = 1$. The equivalent circuit valid under these conditions is shown in Fig: 3.16 (b). Since the current drawn is decided by the resistance and leakage impedances alone, the magnitude can be very high when rated voltage is applied. Therefore in this test, only small voltages are applied — just enough to cause rated current to flow. While the current magnitude depends on the resistance and the reactance, the power drawn depends on the resistances.

The parameters may then be determined as follows. The source current and power drawn may be written as -

$$I_s = \frac{V_s}{(R_s + R'_r) + j(X_s + X'_r)}$$

$$P_s = |I_s|^2(R_s + R'_r)$$
In the test, $V_s$, $I_s$ and $P_s$ are measured with appropriate meters. Above equation enables us to compute \((R_s + R^r)\). Once this is known, \((X_s + X^r)\) may be computed from the above equation.

Note that this test only enables us to determine the series combination of the resistance and the reactance only and not the individual values. Generally, the individual values are assumed to be equal; the assumption \(R_s = R^r\), and \(X_s = X^r\) suffices for most purposes.

In practice, there are differences. If more accurate estimates are required IEEE guidelines may be followed which depend on the size of the machine.

These two tests determine the equivalent circuit parameters in a ‘Stator-referred’ sense, i.e., the rotor resistance and leakage inductance are not the actual values but what they ‘appear to be’ when looked at from the stator. This is sufficient for most purposes as interconnections to the external world are generally done at the stator terminals.

### 3.12.3 Construction of Circle Diagram

Conduct No load test and blocked rotor test on the induction motor and find out the per phase values of no load current $I_0$, short circuit current $I_{SC}$ and the corresponding phase angles $\Phi_0$ and $\Phi_{SC}$. Also find short circuit current $I_{SN}$ corresponding to normal supply voltage. With this data, the circle diagram can be drawn as follows see Fig: 3.17.

1. With suitable scale, draw vector OA with length corresponding to $I_0$ at an angle $\Phi_0$ from the vertical axis. Draw a horizontal line AB.
2. Draw OS equal to $I_{SN}$ at an angle $\Phi_{SC}$ and join AS.
3. Draw the perpendicular bisector to AS to meet the horizontal line AB at C.
4. With C as centre, draw a portion of circle passing through A and S. This forms the circle diagram which is the locus of the input current.
5. From point S, draw a vertical line SL to meet the line AB.
6. Divide SL at point K so that $SK : KL = \text{rotor resistance} : \text{stator resistance}$.
7. For a given operating point P, draw a vertical line PEGD as shown. then $PE = \text{output power}$, $EF = \text{rotor copper loss}$, $FG = \text{stator copper loss}$, $GD = \text{constant loss (iron loss + mechanical loss)}$
8. To find the operating points corresponding to maximum power and maximum torque, draw tangents to the circle diagram parallel to the output line and torque line respectively. The points at which these tangents touch the circle are respectively the maximum power point and maximum torque point.
1. The output line AS is extended backwards to meet the X-axis at O'.

2. From any convenient point on the extended output line, draw a horizontal line QT so as to meet the vertical from O'. Divide the line QT into 100 equal parts.

3. To find the efficiency corresponding to any operating point P, draw a line from O' to the efficiency line through P to meet the efficiency line at T1. Now QT1 is the efficiency.

**Slip Line**

1. Draw line QR parallel to the torque line, meeting the vertical through A at R. Divide RQ into 100 equal parts.

2. To find the slip corresponding to any operating point P, draw a line from A to the slip line through P to meet the slip line at R1. Now RR1 is the slip.

**Power Factor Curve**

1. Draw a quadrant of a circle with O as centre and any convenient radius. Divide OCm into 100 equal parts.

2. To find power factor corresponding to P, extend the line OP to meet the power factor curve at C'. Draw a horizontal line C'C1 to meet the vertical axis at C1. Now OC1 represents power factor.
3.13 Performance Characteristics of Three phase Induction Motor

The equivalent circuits derived in the preceding section can be used to predict the performance characteristics of the induction machine. The important performance characteristics in the steady state are the efficiency, power factor, current, starting torque, maximum (or pull-out) torque.

3.13.1 The complete torque-speed characteristic

In order to estimate the speed-torque characteristic let us suppose that a sinusoidal voltage is impressed on the machine. Recalling that the equivalent circuit is the per-phase representation of the machine, the current drawn by the circuit is given by

\[ I_s = \frac{V_s}{(R_s + R'_r/s) + j(X_{ls} + X'_{lr})} \]

Where, \( V_s \) is the phase voltage phasor and \( I_s \) is the current phasor. The magnetizing current is neglected. Since this current is flowing through \( R'_r/s \), the air-gap power is given by

\[ P_g = \frac{|I_s|^2 R'_r}{s} = \frac{V_s}{(R_s + R'_r/s)^2 + (X_{ls} + X'_{lr})^2} \frac{R'_r}{s} \]

The mechanical power output was shown to be \((1-s)P_g\) (power dissipated in \( R'_r/s \)). The torque is obtained by dividing this by the shaft speed \( \omega_m \). Thus we have,

\[ \frac{P_g(1-s)}{\omega_m} = \frac{P_g(1-s)}{\omega_s(1-s)} = \frac{|I_s|^2 R'_r}{s \omega_s} \]

where \( \omega_m \) is the synchronous speed in radians per second and \( s \) is the slip. Further, this is the torque produced per phase. Hence the overall torque is given by

\[ T_e = \frac{3}{\omega_s} \frac{V_s^2}{(R_s + R'_r/s)^2 + (X_{ls} + X'_{lr})^2} \frac{R'_r}{s} \]

The torque may be plotted as a function of ‘s’ and is called the torque-slip (or torque-speed, since slip indicates speed) characteristic a very important characteristic of the induction machine.

A typical torque-speed characteristic is shown in Fig: 3.18. This plot corresponds to a 3 kW, 4 pole, and 60 Hz machine. The rated operating speed is 1780 rpm.
Further, this curve is obtained by varying slip with the applied voltage being held constant. Coupled with the fact that this is an equivalent circuit valid under steady state, it implies that if this characteristic is to be measured experimentally, we need to look at the torque for a given speed after all transients have died down. One cannot, for example, try to obtain this curve by directly starting the motor with full voltage applied to the terminals and measuring the torque and speed dynamically as it runs up to steady speed.

Fig: 3.18

With respect to the direction of rotation of the air-gap flux, the rotor maybe driven to higher speeds by a prime mover or may also be rotated in the reverse direction. The torque-speed relation for the machine under the entire speed range is called the complete speed-torque characteristic. A typical curve is shown in Fig: 3.19 for a four-pole machine, the synchronous speed being 1500 rpm. Note that negative speeds correspond to slip values greater than 1, and speeds greater than 1500 rpm correspond to negative slip. The plot also shows the operating modes of the induction machine in various regions. The slip axis is also shown for convenience.
3.13.2 Effect of Rotor Resistance on Speed Torque Characteristic

Restricting ourselves to positive values of slip, we see that the curve has a peak point. This is the maximum torque that the machine can produce, and is called as stalling torque. If the load torque is more than this value, the machine stops rotating or stalls. It occurs at a slip \( s^* \), which for the machine of Fig: 3.19 is 0.38. At values of slip lower than \( s^* \), the curve falls steeply down to zero at \( s = 0 \). The torque at synchronous speed is therefore zero. At values of slip higher than \( s = s^* \), the curve falls slowly to a minimum value at \( s = 1 \). The torque at \( s = 1 \) (speed = 0) is called the starting torque. The value of the stalling torque may be obtained by differentiating the expression for torque with respect to zero and setting it to zero to find the value of \( s^* \). Using this method, we can write

\[
\hat{s} = \frac{\pm R'_r}{\sqrt{R'_r^2 + (X_{ls} + X'_{tr})^2}}
\]

Substituting \( \hat{s} \) into the expression for torque gives us the value of the stalling torque \( \hat{T}_e \),

\[
\hat{T}_e = \frac{3V_s^2}{2\omega_s} \cdot \frac{1}{R_s \pm \sqrt{R'_r^2 + (X_{ls} + X'_{tr})^2}}
\]

- The negative sign being valid for negative slip.
The expression shows that \( T_e \) is the independent of \( R' \), while \( \dot{s} \) is directly proportional to \( R' \). This fact can be made use of conveniently to alter \( \dot{s} \). If it is possible to change \( R' \), then we can get a whole series of torque-speed characteristics, the maximum torque remaining constant all the while.

We may note that if \( R' \) is chosen equal to

\[
\sqrt{R_s^2 + (X_{ls} + X'_{tr})^2}
\]

The \( \dot{s} \), becomes unity, which means that the maximum torque occurs at starting. Thus changing of \( R' \), wherever possible can serve as a means to control the starting torque Fig: 3.20.

While considering the negative slip range, (generator mode) we note that the maximum torque is higher than in the positive slip region (motoring mode).

### 3.13.3 Operating Point and Stable & Unstable region of Operation

Consider a speed torque characteristic shown in fig. 25 for an induction machine, having the load characteristic also superimposed on it. The load is a constant torque load i.e. the torque required for operation is fixed irrespective of speed.
The system consisting of the motor and load will operate at a point where the two characteristics meet. From the above plot, we note that there are two such points. We therefore need to find out which of these is the actual operating point. To answer this we must note that, in practice, the characteristics are never fixed; they change slightly with time. It would be appropriate to consider a small band around the curve drawn where the actual points of the characteristic will lie. This being the case let us consider that the system is operating at point 1, and the load torque demand increases slightly. This is shown in Fig: 3.22, where the change is exaggerated for clarity. This would shift the point of operation to a point 1′ at which the slip would be less and the developed torque higher.

The difference in torque developed $\Delta T_e$, being positive will accelerate the machine. Any overshoot in speed as it approaches the point 1′ will cause it to further accelerate since the developed torque is increasing. Similar arguments may be used to show that if for some reason the developed torque becomes smaller the speed would drop and the effect is cumulative. Therefore we may conclude that 1 is not a stable operating point.

Let us consider the point 2. If this point shifts to 2′, the slip is now higher (speed is lower) and the positive difference in torque will accelerate the machine. This behaviour will tend to bring the operating point towards 2 once again. In other words, disturbances at point 2 will not cause a
runaway effect. Similar arguments may be given for the case where the load characteristic shifts down. Therefore we conclude that point 2 is a stable operating point.

![Graph showing speed-torque characteristic](image)

**Fig: 3.22**

From the above discussions, we can say that the entire region of the speed-torque characteristic from \( s = 0 \) to \( s = \hat{s} \) is an unstable region, while the region from \( s = \hat{s} \) to \( s = 0 \) is a stable region. Therefore the machine will always operate between \( s = 0 \) and \( s = \hat{s} \).

### 3.14 Operation with Unbalanced Supply Voltage on Polyphase Induction Motors

Three phase induction motors are designed and manufactured such that all three phases of the winding are carefully balanced with respect to the number of turns, placement of the winding, and winding resistance. When line voltages applied to a polyphase induction motor are not exactly the same, unbalanced currents will flow in the stator winding, the magnitude depending upon the amount of unbalance. A small amount of voltage unbalance may increase the current an excessive amount. The effect on the motor can be severe and the motor may overheat to the point of burnout.
Unbalance Defined

The voltage unbalance (or negative sequence voltage) in percent may be defined as follows:

\[
\text{Percent Voltage Unbalance} = 100 \times \left( \frac{\text{Maximum Voltage Deviation}}{\text{Average Voltage}} \right)
\]

Example:

With voltages of 220, 215 and 210, in three phases respectively then the average is 215, the maximum deviation from the average is 5, and the percent unbalance = \(100 \times \frac{5}{215} = 2.3\) percent.

Effect on performance-

General

The effect of unbalanced voltages on polyphase induction motors is equivalent to the introduction of a "negative sequence voltage" having a rotation opposite to that occurring with balanced voltages. This negative sequence voltage produces in the air gap a flux rotating against the rotation of the rotor, tending to produce high currents. A small negative sequence voltage may produce in the windings currents considerably in excess of those present under balanced voltage conditions.

Temperature rise and load carrying capacity

A relatively small unbalance in voltage will cause a considerable increase in temperature rise. In the phase with the highest current, the percentage increase in temperature rise will be approximately two times the square of the percentage voltage unbalance. The increase in losses and consequently, the increase in average heating of the whole winding will be slightly lower than the winding with the highest current.

To illustrate the severity of this condition, an approximate 3.5 percent voltage unbalance will cause an approximate 25 percent increase in temperature rise.

Torques

The locked-rotor torque and breakdown torque are decreased when the voltage is unbalanced. If the voltage unbalance should be extremely severe, the torque might not be adequate for the application.

Full-load speed

The full-load speed is reduced slightly when the motor operates at unbalanced voltages.
Currents

The locked-rotor current will be unbalanced to the same degree that the voltages are unbalanced but the locked-rotor KVA will increase only slightly. The currents at normal operating speed with unbalanced voltages will be greatly unbalanced in the order of approximately 6 to 10 times the voltage unbalance. This introduces a complex problem in selecting the proper overload protective devices, particularly since devices selected for one set of unbalanced conditions may be inadequate for a different set of unbalanced voltages. Increasing the size of the overload protective device is not the solution in as much as protection against heating from overload and from single phase operation is lost.

Thus the voltages should be evenly balanced as closely as can be read on the usually available commercial voltmeter.

3.15 Starting of Three Phase Induction Motor

The induction motor is fundamentally a transformer in which the stator is the primary and the rotor is short-circuited secondary. At starting, the voltage induced in the induction motor rotor is maximum (s = 1). Since the rotor impedance is low, the rotor current is excessively large. This large rotor current is reflected in the stator because of transformer action. This results in high starting current (4 to 10 times the full-load current) in the stator at low power factor and consequently the value of starting torque is low. Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally.

However, this large starting current will produce large line-voltage drop. This will adversely affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the magnitude of stator current at starting and several methods are available for this purpose.

3.15.1 Methods of Starting Three Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

(i) Direct-on-line starting
(ii) Stator resistance starting
(iii) Autotransformer starting
(iv) Star-delta starting
(v) Rotor resistance starting
Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel-cage motors, depending upon the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

(i) **Direct-on-line starting**

This method of starting in just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

**Relation between starting and F.L. torques.** We know that:

\[
\text{Rotor input} = 2\pi N_s T = kT
\]

But \[\text{Rotor Cu loss} = s \times \text{Rotor input}\]

\[\therefore 3(I'_{2})^2 R_2 = s \times kT\]

or \[T \propto (I'_{2})^2 / s\]

or \[T \propto I_1^2 / s \quad (\because \ I'_2 \propto I_1)\]

If \(I_{st}\) is the starting current, then starting torque \((T_{st})\) is

\[T \propto I_{st}^2 \quad (\because \text{at starting } s = 1)\]

If \(I_f\) is the full-load current and \(s_f\) is the full-load slip, then,

\[T_f \propto I_f^2 / s_f\]

\[\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f\]
When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current $I_{sc}$.

\[ \frac{T_{st}}{T_f} = \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f \]

Let us illustrate the above relation with a numerical example. Suppose $I_{sc} = 5I_f$ and full-load slip $s_f = 0.04$. Then,

\[ \frac{T_{st}}{T_f} = \left( \frac{5I_f}{I_f} \right)^2 \times 0.04 = (5)^2 \times 0.04 = 1 \]

\[ \therefore T_{st} = T_f \]

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

(ii) **Stator resistance starting**

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor see Fig: 3.23.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.

![Fig: 3.23](image-url)
Relation between starting and F.L. torques.

Let $V$ be the rated voltage/phase. If the voltage is reduced by a fraction $x$ by the insertion of resistors in the line, then voltage applied to the motor per phase will be $xV$.

So,

$$I_{st} = x \times I_{sc}$$

Now

$$\frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 \times s_f$$

or

$$\frac{T_{st}}{T_f} = x^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$$

Thus while the starting current reduces by a fraction $x$ of the rated-voltage starting current ($I_{sc}$), the starting torque is reduced by a fraction $x^2$ of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

(iii) Autotransformer starting

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed. Fig: 3.24 shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value. When the motor attains about 80% of normal speed, the changeover switch is thrown to “run” position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.
Relation between starting and F.L. torques. Consider a star-connected squirrel-cage induction motor. If $V$ is the line voltage, then voltage across motor phase on direct switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. In case of autotransformer, if a tapping of transformation ratio $K$ (a fraction) is used, then phase voltage across motor is $KV/\sqrt{3}$ and $I_{st} = K I_{sc}$.

Now

$$T_{st} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f = \left(\frac{K I_{sc}}{I_f}\right)^2 \times s_f = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

$$\therefore T_{st} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$
The current taken from the supply or by autotransformer is \( I_1 = KI_2 = K^2I_{sc} \). Note that motor current is \( K \) times, the supply line current is \( K^2 \) times and the starting torque is \( K^2 \) times the value it would have been on direct-on-line starting.

*(iv) Star-delta starting*

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig: 3.26.

The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to “Start” position which connects the stator windings in star. Therefore, each stator phase gets \( V/\sqrt{3} \) volts where \( V \) is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to “Run” position which connects the stator windings in delta. Now each stator phase gets full line voltage \( V \). The disadvantages of this method are:

(a) With star-connection during starting, stator phase voltage is \( 1/\sqrt{3} \) times the line voltage. Consequently, starting torque is \( \left(1/\sqrt{3}\right)^2 \) or 1/3 times the value it would have with \( \Delta \)-connection. This is rather a large reduction in starting torque.

(b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

**Relation between starting and F.L. torques.** In direct delta starting,

Starting current/phase, \( I_{sc} = V/Z_{sc} \) where \( V \) = line voltage

Starting line current = \( \sqrt{3} I_{sc} \)

In star starting, we have,

Starting current/phase, \( I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}} I_{sc} \)

Now \( \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f = \left(\frac{I_{sc}}{\sqrt{3} \times I_f}\right)^2 \times s_f \)
Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

3.15.2 Starting of Slip-Ring Induction Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig: 3.27.
(i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.

(ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

3.16 Speed control of Three Phase Induction Motors

The induction machine, when operating from mains is essentially a constant speed machine. Many industrial drives, typically for fan or pump applications, have typically constant speed requirements and hence the induction machine is ideally suited for these. However, the induction machine, especially the squirrel cage type, is quite rugged and has a simple construction. Therefore it is good candidate for variable speed applications if it can be achieved.

3.16.1 Speed control by changing applied voltage

From the torque equation of the induction machine we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in Fig: 3.28. These curves show that the slip at maximum torque $S$ remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same.
Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

Fig: 3.28 also shows a load torque characteristic — one that is typical of a fan type of load. In a fan (blower) type of load, the variation of torque with speed is such that $T \propto \omega^2$.

Here one can see that it may be possible to run the motor to lower speeds within the range $n_s$ to $(1 - \hat{s}) n_s$. Further, since the load torque at zero speed is zero, the machine can start even at reduced voltages. This will not be possible with constant torque type of loads.

One may note that if the applied voltage is reduced, the voltage across the magnetising branch also comes down. This in turn means that the magnetizing current and hence flux level are reduced. Reduction in the flux level in the machine impairs torque production which is primarily the explanation for Fig: 3.28. If, however, the machine is running under lightly loaded conditions, then operating under rated flux levels is not required. Under such conditions,
reduction in magnetizing current improves the power factor of operation. Some amount of energy saving may also be achieved.

Voltage control may be achieved by adding series resistors (a lossy, inefficient proposition), or a series inductor / autotransformer (a bulky solution) or a more modern solution using semiconductor devices. A typical solid state circuit used for this purpose is the AC voltage controller or AC chopper.

### 3.16.2 Rotor resistance control

The expression for the torque of the induction machine is dependent on the rotor resistance. Further the maximum value is independent of the rotor resistance. The slip at maximum torque is dependent on the rotor resistance. Therefore, we may expect that if the rotor resistance is changed, the maximum torque point shifts to higher slip values, while retaining a constant torque. Fig: 3.29 shows a family of torque-speed characteristic obtained by changing the rotor resistance.

Note that while the maximum torque and synchronous speed remain constant, the slip at which maximum torque occurs increases with increase in rotor resistance, and so does the starting torque. Whether the load is of constant torque type or fan-type, it is evident that the speed control range is more with this method. Further, rotor resistance control could also be used as a means of generating high starting torque.
For all its advantages, the scheme has two serious drawbacks. Firstly, in order to vary the rotor resistance, it is necessary to connect external variable resistors (winding resistance itself cannot be changed). This, therefore necessitates a slip-ring machine, since only in that case rotor terminals are available outside. For cage rotor machines, there are no rotor terminals. Secondly, the method is not very efficient since the additional resistance and operation at high slips entails dissipation.

The resistors connected to the slip-ring brushes should have good power dissipation capability. Water based rheostats may be used for this. A ‘solid-state’ alternative to a rheostat is a chopper controlled resistance where the duty ratio control of the chopper presents a variable resistance load to the rotor of the induction machine.
3.16.3 Cascade control

The power drawn from the rotor terminals could be spent more usefully. Apart from using the heat generated in meaningful ways, the slip ring output could be connected to another induction machine. The stator of the second machine would carry slip frequency currents of the first machine which would generate some useful mechanical power. A still better option would be to mechanically couple the shafts of the two machines together. This sort of a connection is called cascade connection and it gives some measure of speed control.

Let the frequency of supply given to the first machine be $f_1$, its number poles be $p_1$, and its slip of operation be $S_1$. Let $f_2$, $p_2$ and $S_2$ be the corresponding quantities for the second machine. The frequency of currents flowing in the rotor of the first machine and hence in the stator of the second machine is $S_1f_1$. Therefore $f_2 = S_1f_1$. Since the machines are coupled at the shaft, the speed of the rotor is common for both. Hence, if $n$ is the speed of the rotor in radians,

$$n = \frac{f_1}{p_1}(1 - s_1) = \pm \frac{s_1f_1}{p_2}(1 - s_2).$$

Note that while giving the rotor output of the first machine to the stator of the second, the resultant stator mmf of the second machine may set up an air-gap flux which rotates in the same direction as that of the rotor, or opposes it. This results in values for speed as –

$$n = \frac{f_1}{p_1 + p_2} \text{ or } n = \frac{f_1}{p_1 - p_2} \quad (s_2 \text{ negligible})$$

The latter expression is for the case where the second machine is connected in opposite phase sequence to the first. The cascade connected system can therefore run at two possible speeds.

Speed control through rotor terminals can be considered in a much more general way. Consider the induction machine equivalent circuit of Fig: 3.30, where the rotor circuit has been terminated with a voltage source $E_r$. 

![Fig. 3.30](image-url)
If the rotor terminals are shorted, it behaves like a normal induction machine. This is equivalent to saying that across the rotor terminals a voltage source of zero magnitude is connected. Different situations could then be considered if this voltage source $E_r$ had a non-zero magnitude. Let the power consumed by that source be $P_r$. Then considering the rotor side circuit power dissipation per phase

$$sE_1I_2'\cos\phi_2 = I_2'R_2' + P_r.$$ 

Clearly now, the value of $s$ can be changed by the value of $P_r$. For $P_r = 0$, the machine is like a normal machine with a short circuited rotor. As $P_r$ becomes positive, for all other circuit conditions remaining constant, $s$ increases or in the other words, speed reduces. As $P_r$ becomes negative, the right hand side of the equation and hence the slip decreases. The physical interpretation is that we now have an active source connected on the rotor side which is able to supply part of the rotor copper losses. When $P_r = -I_2'^2R_2'$ the entire copper loss is supplied by the external source. The RHS and hence the slip is zero. This corresponds to operation at synchronous speed. In general the circuitry connected to the rotor may not be a simple resistor or a machine but a power electronic circuit which can process this power requirement. This circuit may drive a machine or recover power back to the mains. Such circuits are called static Kramer drives.

### 3.16.4 Pole changing method

Sometimes induction machines have a special stator winding capable of being externally connected to form two different number of pole numbers. Since the synchronous speed of the induction machine is given by $n_s = f_s/2p$ (in rev./s) where $p$ is the number of pole pairs, this would correspond to changing the synchronous speed. With the slip now corresponding to the new synchronous speed, the operating speed is changed. This method of speed control is a stepped variation and generally restricted to two steps.

If the changes in stator winding connections are made so that the air gap flux remains constant, then at any winding connection, the same maximum torque is achievable. Such winding arrangements are therefore referred to as constant-torque connections. If however such connection changes result in air gap flux changes that are inversely proportional to the synchronous speeds, then such connections are called constant-horsepower type.

The following figure serves to illustrate the basic principle. Consider a magnetic pole structure consisting of four pole faces A, B, C, D as shown in Fig: 3.31.
Coils are wound on A & C in the directions shown. The two coils on A & C may be connected in series in two different ways — A2 may be connected to C1 or C2. A1 with the other terminal at C then form the terminals of the overall combination. Thus two connections result as shown in Fig: 3.32 (a) & (b).

Now, for a given direction of current flow at terminal A1, say into terminal A1, the flux directions within the poles are shown in the figures. In case (a), the flux lines are out of the pole A (seen from the rotor) for and into pole C, thus establishing a two-pole structure. In case (b) however, the flux lines are out of the poles in A & C. The flux lines will be then have to complete the circuit by flowing into the pole structures on the sides. If, when seen from the rotor, the pole emanating flux lines is considered as North Pole and the pole into which they enter is termed as south, then the pole configurations produced by these connections is a two-pole arrangement in Fig: 3.32(a) and a four-pole arrangement in Fig: 3.32 (b).
Thus by changing the terminal connections we get either a two pole air-gap field or a four-pole field. In an induction machine this would correspond to a synchronous speed reduction in half from case (a) to case (b). Further note that irrespective of the connection, the applied voltage is balanced by the series addition of induced emfs in two coils. Therefore the air-gap flux in both cases is the same. Cases (a) and (b) therefore form a pair of constant torque connections.

Consider, on the other hand a connection as shown in the Fig: 3.32 (c). The terminals T1 and T2 are where the input excitation is given. Note that current direction in the coils now resembles that of case (b), and hence this would result in a four-pole structure. However, in Fig: 3.32 (c), there is only one coil induced emf to balance the applied voltage. Therefore flux in case (c) would therefore be halved compared to that of case (b) or case (a), for that matter. Cases (a) and (c) therefore form a pair of constant horse-power connections.

It is important to note that in generating a different pole numbers, the current through one coil (out of two, coil C in this case) is reversed. In the case of a three phase machine, the following example serves to explain this. Let the machine have coils connected as shown [C1 – C6] as shown in Fig: 3.33.
The current directions shown in C1 & C2 correspond to the case where T1, T2, T3 are supplied with three phase excitation and Ta, Tb & Tc are shorted to each other (STAR point). The applied voltage must be balanced by induced emf in one coil only (C1 & C2 are parallel). If however the excitation is given to Ta, Tb & Tc with T1, T2, T3 open, then current through one of the coils (C1 & C2) would reverse. Thus the effective number of poles would increase, thereby bringing down the speed. The other coils also face similar conditions.

![Diagram](image.png)

Fig: 3.33

3.16.5 Stator frequency control

The expression for the synchronous speed indicates that by changing the stator frequency also it can be changed. This can be achieved by using power electronic circuits called inverters which convert dc to ac of desired frequency. Depending on the type of control scheme of the inverter, the ac generated may be variable-frequency-fixed-amplitude or variable-frequency-variable-amplitude type. Power electronic control achieves smooth variation of voltage and frequency of the ac output. This when fed to the machine is capable of running at a controlled speed. However, consider the equation for the induced emf in the induction machine.

\[ V = 4.44N\phi_m f \]

Where, N is the number of the turns per phase, \( \phi_m \) is the peak flux in the air gap and \( f \) is the frequency.
Note that in order to reduce the speed, frequency has to be reduced. If the frequency is reduced while the voltage is kept constant, thereby requiring the amplitude of induced emf to remain the same, flux has to increase. This is not advisable since the machine likely to enter deep saturation. If this is to be avoided, then flux level must be maintained constant which implies that voltage must be reduced along with frequency. The ratio is held constant in order to maintain the flux level for maximum torque capability.

Actually, it is the voltage across the magnetizing branch of the exact equivalent circuit that must be maintained constant, for it is that which determines the induced emf. Under conditions where the stator voltage drop is negligible compared the applied voltage. In this mode of operation, the voltage across the magnetizing inductance in the ‘exact’ equivalent circuit reduces in amplitude with reduction in frequency and so does the inductive reactance. This implies that the current through the inductance and the flux in the machine remains constant. The speed torque characteristics at any frequency may be estimated as before. There is one curve for every excitation frequency considered corresponding to every value of synchronous speed. The curves are shown below. It may be seen that the maximum torque remains constant.

This may be seen mathematically as follows. If $E$ is the voltage across the magnetizing branch and $f$ is the frequency of excitation, then $E = kf$, where $k$ is the constant of proportionality. If $f = 2\pi f$, the developed torque is given by
If this equation is differentiated with respect to \( s \) and equated to zero to find the slip at maximum torque \( \hat{s} \), we get \( \hat{s} = \pm R'_r/(\omega L'_{lr}) \). The maximum torque is obtained by substituting this value into above equation,

\[
\hat{T}_{E/f} = \frac{k^2 f^2}{(R'_s/s)^2 + (\omega L'_{lr})^2} \frac{R'_r}{s\omega} \]

It shows that this maximum value is independent of the frequency. Further \( s\omega \) is independent of frequency. This means that the maximum torque always occurs at a speed lower than synchronous speed by a fixed difference, independent of frequency. The overall effect is an apparent shift of the torque-speed characteristic as shown in Fig: 3.34.

Though this is the aim, \( E \) is an internal voltage which is not accessible. It is only the terminal voltage \( V \) which we have access to and can control. For a fixed \( V \), \( E \) changes with operating slip (rotor branch impedance changes) and further due to the stator impedance drop. Thus if we approximate \( E/f \) as \( V/f \), the resulting torque-speed characteristic shown in Fig: 3.35 is far from desirable.

![Fig: 3.35](image-url)
At low frequencies and hence low voltages the curves show a considerable reduction in peak torque. At low frequencies (and hence at low voltages) the drop across the stator impedance prevents sufficient voltage availability. Therefore, in order to maintain sufficient torque at low frequencies, a voltage more than proportional needs to be given at low speeds.

Another component of compensation that needs to be given is due to operating slip. With these two components, therefore, the ratio of applied voltage to frequency is not a constant but is a curve such as that shown in Fig: 3.36

![Fig: 3.36](image)

With this kind of control, it is possible to get a good starting torque and steady state performance. However, under dynamic conditions, this control is insufficient. Advanced control techniques such as field- oriented control (vector control) or direct torque control (DTC) are necessary.

### 3.17 Power Stages in an Induction Motor

The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are:

**1. Fixed losses**

(i) Stator iron loss
(ii) Friction and windage loss

The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

2. Variable losses

(i) Stator copper loss
(ii) Rotor copper loss

Fig: 3.37 shows how electric power fed to the stator of an induction motor suffers losses and finally converted into mechanical power.

![Diagram showing power losses in an induction motor]

Fig: 3.37

The following points may be noted from the above diagram:

(i) Stator input, $P_i = \text{Stator output} + \text{Stator losses}$

$= \text{Stator output} + \text{Stator Iron loss} + \text{Stator Cu loss}$

(ii) Rotor input, $P_r = \text{Stator output}$

It is because stator output is entirely transferred to the rotor through air-gap by electromagnetic induction.
(iii) Mechanical power available, $P_m = P_r - \text{Rotor Cu loss}$

This mechanical power available is the gross rotor output and will produce a gross torque $T_g$.

(iv) Mechanical power at shaft, $P_{out} = P_m - \text{Friction and windage loss}$

Mechanical power available at the shaft produces a shaft torque $T_{sh}$.

Clearly, $P_m - P_{out} = \text{Friction and windage loss}$.

### 3.18 Double Cage Induction Motor

One of the advantages of the slip-ring motor is that resistance may be inserted in the rotor circuit to obtain high starting torque (at low starting current) and then cut out to obtain optimum running conditions. However, such a procedure cannot be adopted for a squirrel cage motor because its cage is permanently short-circuited. In order to provide high starting torque at low starting current, double-cage construction is used.

**Construction**

As the name suggests, the rotor of this motor has two squirrel-cage windings located one above the other as shown in Fig: 3.38(i).

*The outer winding* consists of bars of smaller cross-section short-circuited by end rings. Therefore, the resistance of this winding is high. Since the outer winding has relatively open slots and a poorer flux path around its bars [See Fig: 3.38(ii)], it has a low inductance. Thus the resistance of the outer squirrel-cage winding is high and its inductance is low.

*The inner winding* consists of bars of greater cross-section short-circuited by end rings. Therefore, the resistance of this winding is low. Since the bars of the inner winding are thoroughly buried in iron, it has a high inductance [See Fig: 3.38(ii)]. Thus the resistance of the inner squirrel cage winding is low and its inductance is high.
Working

When a rotating magnetic field sweeps across the two windings, equal e.m.f.s are induced in each.

(i) At starting, the rotor frequency is the same as that of the line (i.e., 50 Hz), making the reactance of the lower winding much higher than that of the upper winding. Because of the high reactance of the lower winding, nearly all the rotor current flows in the high-resistance outer cage winding. This provides the good starting characteristics of a high-resistance cage winding. Thus the outer winding gives high starting torque at low starting current.

(ii) As the motor accelerates, the rotor frequency decreases, thereby lowering the reactance of the inner winding, allowing it to carry a larger proportion of the total rotor current. At the normal operating speed of the motor, the rotor frequency is so low (2 to 3 Hz) that nearly all the rotor current flows in the low-resistance inner cage winding. This results in good operating efficiency and speed regulation.
Fig: 3.39 shows the operating characteristics of double squirrel-cage motor. The starting torque of this motor ranges from 200 to 250 percent of full-load torque with a starting current of 4 to 6 times the full-load value. It is classed as a high-torque, low starting current motor.

3.19 Cogging and Crawling of Induction Motor

Crawling of induction motor

Sometimes, squirrel cage induction motors exhibits a tendency to run at very slow speeds (as low as one-seventh of their synchronous speed). This phenomenon is called as crawling of an induction motor.

This action is due to the fact that, flux wave produced by a stator winding is not purely sine wave. Instead, it is a complex wave consisting a fundamental wave and odd harmonics like 3rd, 5th, 7th etc. The fundamental wave revolves synchronously at synchronous speed $N_s$ whereas 3rd, 5th, 7th harmonics may rotate in forward or backward direction at $N_s/3$, $N_s/5$, $N_s/7$ speeds respectively. Hence, harmonic torques are also developed in addition with fundamental torque.

3rd harmonics are absent in a balanced 3-phase system. Hence 3rd harmonics do not produce rotating field and torque. The total motor torque now consist three components as: (i) the fundamental torque with synchronous speed $N_s$, (ii) 5th harmonic torque with synchronous speed
Ns/5, (iv) 7th harmonic torque with synchronous speed Ns/7 (provided that higher harmonics are neglected).

Now, 5th harmonic currents will have phase difference of

\[ 5 \times 120^\circ = 600^\circ = 2 \times 360^\circ - 120^\circ = -120^\circ. \]

Hence the revolving speed set up will be in reverse direction with speed Ns/5. The small amount of 5th harmonic torque produces breaking action and can be neglected.

The 7th harmonic currents will have phase difference of

\[ 7 \times 120^\circ = 840^\circ = 2 \times 360^\circ + 120^\circ = +120^\circ. \]

Hence they will set up rotating field in forward direction with synchronous speed equal to Ns/7. If we neglect all the higher harmonics, the resultant torque will be equal to sum of fundamental torque and 7th harmonic torque. 7th harmonic torque reaches its maximum positive value just before 1/7th of Ns. If the mechanical load on the shaft involves constant load torque, the torque developed by the motor may fall below this load torque. In this case, motor will not accelerate up to its normal speed, but it will run at a speed which is nearly 1/7th of its normal speed as shown in Fig: 3.40. This phenomenon is called as crawling of induction motors.

Fig: 3.40
Cogging (Magnetic Locking or Teeth Locking) of induction motor

Sometimes, the rotor of a squirrel cage induction motor refuses to start at all, particularly if the supply voltage is low. This happens especially when the number of rotor teeth is equal to the number of stator teeth, because of magnetic locking between the stator teeth and the rotor teeth. When the rotor teeth and stator teeth face each other, the reluctance of the magnetic path is minimum that is why the rotor tends to remain fixed. This phenomenon is called cogging or magnetic locking of induction motor.

3.20 Induction Generator

When a squirrel cage induction motor is energized from a three-phase power system and is mechanically driven above its synchronous speed, it will deliver power to the system. An induction generator receives its excitation (magnetizing current) from the system to which it is connected. It consumes rather than supplies reactive power (KVAR) and supplies only real power (KW) to the system. The KVAR required by the induction generator plus the KVAR requirements of all other loads on the system must be supplied from synchronous generators or static capacitors on the system.

Operating as a generator at a given percentage slip above synchronous speed, the torque, current, efficiency and power factor will not differ greatly from that when operating as a motor. The same slip below synchronous speed, the shaft torque and electric power flow is reversed. Typical speed torque characteristic of induction generator is shown in Fig: 3.41.
Now for example, a 3600 RPM squirrel cage induction motor which delivers full load output at 3550 RPM as a motor will deliver full rated power as a generator at 3650 RPM. If the half-load motor speed is 3570 RPM, the output as a generator will be one-half of rated value when driven at 3630 RPM, etc. Since the induction generator is actually an induction motor being driven by a prime mover, it has several advantages.

1. It is less expensive and more readily available than a synchronous generator.
2. It does not require a DC field excitation voltage.
3. It automatically synchronizes with the power system, so its controls are simpler and less expensive.

The principal disadvantages of an induction generator are listed below

1. It is not suitable for separate, isolated operation
2. It consumes rather than supplies magnetizing KVAR
3. It cannot contribute to the maintenance of system voltage levels (this is left entirely to the synchronous generators or capacitors)
4. In general it has a lower efficiency.

**Induction Generator Application**

As energy costs so high, energy recovery became an important part of the economics of most industrial processes. The induction generator is ideal for such applications because it requires very little in the way of control system or maintenance.

Because of their simplicity and small size per kilowatt of output power, induction generators are also favoured very strongly for small windmills. Many commercial windmills are designed to operate in parallel with large power systems, supplying a fraction of the customer’s total power needs. In such operation, the power system can be relied on for voltage & frequency control, and static capacitors can be used for power-factor correction.
MODULE-IV

SINGLE PHASE MOTORS

SYLLABUS/ TOPICS COVERED

Three Phase Induction Motors: Types, Construction and principle of operation, 3 phase Induction Motor, general phasor diagram, equivalent circuit, power and torque relations, condition for maximum torque, circle diagram, Performance characteristics, effect of rotor resistance on speed torque characteristics, stable & unstable region of operation, Operation with unbalanced supply voltage. Starting: Starting of 3 phase induction motors, high starting torque motors, speed control, rheostatic method, pole changing method cascade control of speed, Double cage induction motor, Cogging and Crawling of Induction motor, induction generator

[Topics are arranged as per above sequence]
Module -IV

4. Single Phase Induction Motors

Single phase induction motors perform a great variety of useful services at home, office, farm, factory and in business establishments. Single phase motors are generally manufactured in fractional HP ratings below 1 HP for economic reasons. Hence, those motors are generally referred to as fractional horsepower motors with a rating of less than 1 HP. Most single phase motors fall into this category. Single phase induction motors are also manufactured in the range of 1.5, 2, 3 and up to 10 HP as a special requirement.

4.1 Theory of Operation

A single phase induction motor is similar in construction to that of a polyphase induction motor with difference that its stator has only one winding. If such a stator is supplied with single phase alternating current, the field produced by it changes in magnitude and direction sinusoidally. Thus the magnetic field produced in the air gap is alternating one but not rotating as a result these kind of motors are NOT SELF STARTING. Fig: 4.2 (a) shows the torque-speed characteristic of single phase induction motor.
Such an alternating field is equivalent to two fields of equal magnitude rotating in opposite directions at equal speed as explained below:

**4.1.1 Double Revolving Field Theory of Single Phase Induction Motor**

Consider two magnetic fields represented by quantities OA and OB of equal magnitude revolving in opposite directions as shown in fig: 4.1.

![Fig: 4.2 (b)]

The resultant of the two fields of equal magnitude rotating in opposite directions is alternating. Therefore an alternating current can be considered as having two components which are of equal in magnitude and rotating in opposite directions.

From the above, it is clear that when a single phase alternating current is supplied to the stator of a single phase motor, the field produced will be of alternating in nature which can be divided into two components of equal magnitude one revolving in clockwise and other in counter clockwise direction.

If a stationary squirrel cage rotor is kept in such a field equal forces in opposite direction will act and the rotor will simply vibrate and there will be no rotation.

But if the rotor is given a small jerk in any direction in this condition, it will go on revolving and will develop torque in that particular direction. It is clear from the above that a single phase induction motor when having only one winding is not a self-starting. To make it a self-starting anyone of the following can be adopted.

(i) Split phase starting.
(ii) Repulsion starting.
(iii) Shaded pole starting.

**4.2 Equivalent Circuit of Single Phase Induction Motor**

The equivalent circuit of single phase induction motor is shown below (Fig: 4.3)
4.2.1 Determination of Equivalent Circuit Parameters of Single Phase Induction motor

It is possible to find the parameters of the equivalent circuit of the single phase induction motor experimentally as shown in Fig.4.4. For this purpose, three tests should be conducted:

1- The DC Test:

The DC resistance of the stator can be measured by applying DC current to the terminals of the main winding and taking the reading of the voltage and the current (or using ohmmeter) and determine the DC resistance as follows:

\[ R_{DC} = \frac{V_{DC}}{I_{DC}} \]

Then, the AC resistance is given by:

\[ R_{AC} = 1.25 \times R_{DC} \]

2-The Blocked Rotor Test:

When the rotor is locked (i.e. prevented from running), \( S_b = S_f = 1 \). The secondary impedances become much less than the magnetizing branches and the corresponding equivalent circuit becomes that of Fig: 4.5.
Fig: 4.4 Equivalent circuit of single phase induction motor.

Fig: 4.5(a) Approximate equivalent circuit of the single phase induction motor at standstill.
The circuit in Fig: 4.5 (a) can be rearranged to the equivalent circuit that is shown in Fig: 4.5(b).

Fig: 4.5(b) Rearranged approximate equivalent circuit of the single phase induction motor at standstill.

The readings to be obtained from this test are:

a) Single phase power $P_{BL}$

b) Phase voltage $V_{BL}$

c) Phase current $I_{BL}$

Then, $R_{eq}$, $Z_{eq}$, and $X_{eq}$ can be obtained using the following equations:

$$R_{eq} = \frac{P_{BL}}{I_{BL}^2}$$

$$Z_{eq} = \frac{V_{BL}}{I_{BL}}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

Separation of $X_1$, $X'_{eq}$, $R_1$, and $R'_2$ can be done as follows:

$$X_1 = X'_{eq} = \frac{1}{2} X_{eq}$$

$$R'_2 = R_{eq} - R_1$$
3-The No Load Test:

When the induction motor is allowed to run freely at no load, the forward slip $S_f$ approaches zero and the backward slip $S_b$ approaches 2 ($S_f = s, S_b = 2 - s$). The secondary forward impedance becomes very large with respect to the magnetizing branch, while the secondary backward impedance becomes very small if compared with the magnetizing branch. Accordingly, the equivalent circuit corresponding to these operating conditions can be approximated by that of Fig: 4.6.

The readings to be obtained from this test are:

d) Single phase power $P_{NL}$

e) Phase voltage $V_{NL}$

f) Phase current $I_{NL}$

![Approximate equivalent circuit of the single phase induction motor at no load.](image-url)
The circuit in Fig: 4.6 (a) can be rearranged to the equivalent circuit that is shown in Fig: 4.6 (b).

![Rearranged approximate equivalent circuit of the single phase induction motor at no load](image)

**Fig: 4.6 (b) Rearranged approximate equivalent circuit of the single phase induction motor at no load**

Then, \( R_w \) and \( X_m \), can be obtained as follows:

\[
P_{\text{core+mechanical}} = P_{NL} - I_{NL}^2 \left( R_1 + \frac{R'_2}{4} \right)
\]

\[
E_{1F} = \bar{V}_{NL} - \bar{I}_{NL} \left( R_1 + \frac{R'_2}{4} \right) + j \left( X_1 + \frac{X'_2}{2} \right)
\]

Note: \( \bar{I}_{NL} = I_{NL} \angle -\theta \), \( \theta = \cos^{-1} \left( \frac{P_{NL}}{V_{NL} I_{NL}} \right) \)

\[
R_w = 2 \left( \frac{|E_{1F}|^2}{P_{\text{core+mechanical}}} \right)
\]

\[
I_w = \frac{|E_{1F}|}{\left( \frac{R_w}{2} \right)} = 2 \frac{|E_{1F}|}{R_w}
\]
4.3 Methods of Starting

It is clear from previous discussion that a single phase induction motor when having only one winding and it is not self-starting. To make it a self-starting anyone of the following can be adopted.

(1) Split phase starting.
(2) Repulsion starting.
(3) Shaded pole starting.

4.3.1 PRINCIPLE OF SPLIT PHASE INDUCTION MOTOR

The basic principle of operation of a split phase induction motor is similar to that of a polyphase induction motor. The main difference is that the single phase motor does not produce a rotating magnetic field but produces only a pulsating filed.

Hence, to produce the rotating magnetic field for self-starting, phase splitting is to be done to make the motor to work as a two phase motor for starting.

4.3.1 Working of Split Phase Motor

In split phase motor two windings named as main winding and starting winding are provided. At the time of starting, both the main and starting windings should be connected across the supply to produce the rotating magnetic field.

The rotor is of a squirrel cage type and the revolving magnetic field sweeps part the stationary rotor, inducing emf in the rotor. As the rotor bars are short-circuited, a current flows through them producing a magnetic field.

This magnetic field opposes the revolving magnetic field and will combine with the main filed to produce a revolving filed. By this action, the rotor starts revolving in the same direction of the rotating magnetic field as in the case of a squirrel cage induction motor.

Hence, once the rotor starts rotating, the starting winding can be disconnected from the supply by some mechanical means as the rotor and stator fields from a revolving magnetic field. There are several types of split phase motors.
4.3.2 TYPES OF SPLIT-PHASE INDUCTION MOTORS

1. Resistance-start, induction-run motors
2. Capacitor-start, induction-run motors
3. Capacitor-start, capacitor-run motors
4. Shaded pole motors.

1. RESISTANCE-START, INDUCTION-RUN MOTORS

As the starting torque of this type of motor is relatively small and its starting current is high, these motors are most commonly used for rating up to 0.5 HP where the load could be started easily. The essential parts are shown in Fig: 4.7.

- Main winding or running winding.
- Auxiliary winding or starting winding
- Squirrel cage type rotor.
- Centrifugal switch.

CONSTRUCTION AND WORKING

The starting winding is designed to have a higher resistance and lower reactance than the main winding. This is achieved by using small conductors in the auxiliary winding than in the main winding. The main winding will have higher inductance when surrounded by more iron, which could be made possible by placing it deeper into the stator slots, it is obvious that the current would split as shown in Fig: 4.7(b).

![Diagram](image)

**Fig: 4.7**
The starting current "I" start will lag the main supply voltage "V" line by 15 degree and the main winding current. "I" main lags the main voltage by about 80 degree. Therefore, these currents will differ in time phase and their magnetic fields will combine to produce a rotating magnetic field.

When the motor has come up to about 75 to 80% of synchronous speed, the starting winding is opened by a centrifugal switch and the motor will continue to operate as a single phase motor.

CHARACTERISTICS

At the point where the starting winding is disconnected, the motor develops nearly as much torque with the main winding alone as with both windings connected. This can be observed from the typical torque-speed characteristics of this motor, as shown in Fig: 4.8.

![Torque-speed characteristics](image)

The direction of rotating of a split-phase motor is determined by the way the main and auxiliary windings are connected. Hence, either by changing the main winding terminals or by changing the starting winding terminals, the reversal of direction of rotating could be obtained.

APPLICATIONS

These motors are used for driving fans, grinders, washing machines.
2. CAPACITOR-START, INDUCTION-RUN MOTOR

A drive which requires a large starting torque may be fitted with a capacitor-start, induction-run motor as it has excellence starting torque as compared to the resistance-start, induction-run motor.

CONSTRUCTION AND WORKING

Fig: 4.9(a) shows the schematic diagram of a capacitor-start, induction-run motor. As shown, the main winding is directly connected across the main supply whereas the starting winding is connected across the main supply through a capacitor and centrifugal switch.

Both these windings are placed in a stator slot at 90 degree electrical apart, and a squirrel cage type rotor is used.

As shown in Fig: 4.9(b), at the time of starting the current in the main winding lags the supply voltages by 90 degrees, depending upon its inductance and resistance. On the other hand, the current in the starting winding due to its capacitor will lead the applied voltage, by say 20 degrees.

Hence, the phase difference between the main and starting winding becomes near to 90 degrees. This in turn makes the line current to be more or less in phase with its applied voltage, making the power factor to be high, thereby creating an excellent starting torque.

However, after attaining 75% of the rated speed, the centrifugal switch operates opening the starting winding and the motor then operates as an induction motor, with only the main winding connected to the supply.

As shown in Fig: 4.9(b), the displacement of current in the main and starting winding is about 80/90 degrees, and the power factor angle between the applied voltage and line current is very small. This results in producing a high power factor and an excellent starting torque, several times higher than the normal running torque as shown in Fig: 4.10.
CHARACTERISTICS

The torque-speed characteristics of this motor is shown in Fig: 4.10.

![Torque-speed characteristics diagram]

Fig: 4.10

In order to reverse the direction of rotation of the capacitor-start, induction-run motor, either the starting or the main winding terminals should be changed.

This is due to the fact that the direction of rotation depends upon the instantaneous polarities of the main field flux and the flux produced by the starting winding. Therefore, reversing the polarity of one of the field will reverse the torque.

APPLICATIONS

Due to the excellent starting torque and easy direction-reversal characteristics,

- Used in belted fans,
- Used in blowers dryers,
- Used in washing machines,
- Used in pumps and compressors.

3. CAPACITOR-START, CAPACITOR-RUN MOTORS

As discussed earlier, one capacitor-start, induction-run motors have excellent starting torque, say about 300% of the full load torque and their power factor during starting in high.

However, their running torque is not good, and their power factor, while running is low. They also have lesser efficiency and cannot take overloads.

CONSTRUCTION AND WORKING

The aforementioned problems are eliminated by the use of a two valve capacitor motor in which one large capacitor of electrolytic (short duty) type is used for starting whereas a smaller
capacitor of oil filled (continuous duty) type is used for running, by connecting them with the starting winding as shown in Fig:4.11. A general view of such a two valve capacitor motor is shown in Fig: 4.11.

![Fig: 4.11](image)

This motor also works in the same way as a capacitor-start, induction-run motor, with exception, that the capacitor C1 is always in the circuit, altering the running performance to a great extent.

The starting capacitor which is of short duty rating will be disconnected from the starting winding with the help of a centrifugal switch, when the starting speed attains about 75% of the rated speed.

CHARACTERISTICS

The torque-speed characteristics of this motor is shown in Fig: 4.12.

![Fig: 4.12](image)
This motor has the following advantages:

- The starting torque is 300% of the full load torque
- The starting current is low, say 2 to 3 times of the running current.
- Starting and running power factor are good.
- Highly efficient running.
- Extremely noiseless operation.
- Can be loaded up to 125% of the full load capacity.

APPLICATIONS

- Used for compressors, refrigerators, air-conditioners, etc.
- Higher starting torque.
- High efficiency, higher power factor and overloading.
- Costlier than the capacitor-start — Induction run motors of the same capacity.

4.3.2 REPULSION STARTING

This type of starting need a wound rotor with brush and commutator arrangement like a DC armature Fig 4.13(a). The starting operation is based on the principle of repulsion and hence the name.

CONSTRUCTION AND WORKING

Repulsion starting, though complicated in construction and higher in cost, are still used in certain industries due to their excellent starting torque, low starting current, ability to withstand long spell of starting currents to drive heavy loads and their easy method of reversal of direction.
Now there is a condition that the rotor north pole will be repelled by the main north pole and the rotor south pole is repelled by the main south pole, so that a torque could be developed in the rotor. Now due to the repulsion action between the stator and the rotor poles, the rotor will start rotating in a clockwise direction. As the motor torque is due to repulsion action, this starting method is named as repulsion starting.

![Diagram of repulsion motor](image)

Fig: 4.13

To change the direction of rotation of this motor, the brush axis needs to be shifted from the right side as shown in Fig:4.13(b) to the left side of the main axis in a counter clockwise direction as shown in Fig:4.13(b).

CHARACTERISTICS

The torque developed in a repulsion motor will depend upon the amount of brush shaft as shown in Fig: 4.13 (b), whereas the direction of shift decides the direction of rotation.

Further, the speed depends upon the amount of brush shift and the magnitude of the load also on the relationship between the torque and brush-position angle.

Though the starting torque from 250 to 400% of the full load torque, the speed will be dangerously high during light loads. This is due to the fact that the speed of the repulsion motor start does not depend on frequency or number of poles but depends upon the repulsion principle.
Further, there is a tendency of sparking in the brushes at heavy loads, and the PF will be poor at low speeds. Hence the conventional repulsion motor start is not much popular.

4.3.3 SHAPED POLE STARTING

The motor consists of a yoke to which salient poles are fitted as shown in Fig: 4.14(a) and it has a squirrel cage type rotor.

![Diagram of motor components](image)

A shaded pole made of laminated sheets has a slot cut across the lamination at about one third the distance from the edge of the pole.

Around the smaller portion of the pole, a short-circuited copper ring is placed which is called the shading coil, and this part of the pole is known as the shaded part of the pole. The remaining part of the pole is called the unshaded part which is clearly shown in Fig: 4.14(b).

Around the poles, exciting coils are placed to which an AC supply is connected. When AC supply is effected to the exciting coil, the magnetic axis shifts from the unshaded part of the pole to the shaded part as will be explained in details in the next paragraph. This shifting of axis is equivalent to the physical movement of the pole.
This magnetic axis, which is moving, cuts the rotor conductors and hence, a rotational torque is developed in the rotor.

By this torque the rotor starts rotating in the direction of the shifting of the magnetic axis that is from the unshaded part to the shaded part.

THE MAGNETIC FLUX SHIFTING

As the shaded coil is of thick copper, it will have very low resistance but as it is embedded in the iron case, it will have high inductance. When the exciting winding is connected to an AC supply, a sine wave current passes through it.

Let us consider the positive half cycle of the AC current as shown in Fig: 4.15.

When the current raises from "Zero" Value of point "0" to a point "a" the change in current is very rapid (Fast). Hence, it reduces an emf in the shaded coil on the basis of Faraday's law of electromagnetic induction.

The induced emf in the shaded coil produces a current which, in turn, produces a flux in accordance with Lenz Law. This induced flux opposes the main flux in the shaded portion and reduces the main flux in that area to a minimum value as shown in Fig: 4.15.

This makes the magnetic axis to be in the centre of the unshaded portion as shown by the arrow in part of Fig: 4.15. On the other hand as shown in part 2 of 3 when the current raises from point "a" to point "b" the change in current is slow the induced emf and resulting current in the shading coil is minimum and the main flux is able to pass through the shade portion.

This makes the magnetic axis to be shifted to the centre of the whole pole as shown in by the arrow in part 2 of Fig: 4.15.
In the next instant, as shown in part 3 of Fig: 4.15. When the current falls from "b" to "c" the change in current is fast but the change of current is from maximum to minimum.

Hence a large current is induced in the shading ring which opposes the diminishing main flux, thereby increasing the flux density in the area of the shaded part. This makes the magnetic axis to shift to the right portion of the shaded part as shown by the arrow in part.

From the above explanation it is clear the magnetic axis shifts from the unshaded part to the shaded part which is more or less a physical rotary movement of the poles.

Simple motors of this type cannot be reversed. Specially designed shaded pole motors have been constructed for reversing operations. Two such types:

a. The double set of shading coils method

b. The double set of exciting winding method.

Shaded pole motors are built commercially in very small sizes, varying approximately from 1/250 HP to 1/6 HP. Although such motors are simple in construction and cheap, there are certain disadvantages with these motor as stated below:

- Low starting torque.
- Very little overload capacity.
- Low efficiency.

APPLICATIONS

- Record players
- Fans
- Hair driers.

4.4 Single Phase Series Motor

The single-phase series motor is a commutator-type motor. If the polarity of the line terminals of a dc series motor is reversed, the motor will continue to run in the same direction. Thus, it might be expected that a dc series motor would operate on alternating current also. The direction of the torque developed in a dc series motor is determined by both filed polarity and the direction of current through the armature $[T \propto \phi I_a]$. 
4.4.1 Operation

Let a dc series motor be connected across a single-phase ac supply. Since the same current flows through the field winding and the armature, it follows that ac reversals from positive to negative, or from negative to positive, will simultaneously affect both the field flux polarity and the current direction through the armature. This means that the direction of the developed torque will remain positive, and rotation will continue in the same direction. Thus, a series motor can run both on dc and ac.

However, a series motor which is specifically designed for dc operation suffers from the following drawbacks when it is used on single-phase ac supply:

1. Its efficiency is low due to hysteresis and eddy-current losses.
2. The power factor is low due to the large reactance of the field and the armature winding.
3. The sparking at the brushes is excessive.

In order to overcome these difficulties, the following modifications are made in a D.C. series motor that is to operate satisfactorily on alternating current:

1. The field core is constructed of a material having low hysteresis loss. It is laminated to reduce eddy-current loss.
2. The field winding is provided with small number of turns. The field-pole areas is increased so that the flux density is reduced. This reduces the iron loss and the reactive voltage drop.
3. The number of armature conductors is increased in order to get the required torque with the low flux.
4. In order to reduce the effect of armature reaction, thereby improving commutation and reducing armature reactance, a compensating winding is used.

The compensating winding is put in the stator slots. The axis of the compensating winding is 90 (electrical) with the main field axis. It may be connected in series with both the armature and field as shown in Fig: 4.16. In such a case the motor is conductively compensated.

The compensating winding may be short circuited on itself, in which case the motor is said to be inductively compensated shown in Fig: 4.17.
The characteristics of single-phase series motor are very much similar to those of D.C. series motors, but the series motor develops less torque when operating from an a.c. supply than when working from an equivalent D.C. supply [Fig: 4.18]. The direction of rotation can be changed by interchanging connections to the field with respect to the armature as in D.C. series motor.
Speed control of universal motors is best obtained by solid-state devices. Since the speed of these is not limited by the supply frequency and may be as high as 20,000 r.p.m. (greater than the maximum synchronous speed of 3000 r.p.m. at 50 Hz), they are most suitable for applications requiring high speeds.

4.4.2 Phasor Diagram of A.C Series Motor

The schematic diagram and phasor diagram for the conductively coupled single-phase ac series motor are shown in Fig: 4.19 and Fig: 4.20 respectively.
The resistance $I_aR_{se}$, $I_aR_i$, $I_aR_c$ and $I_aR_a$ drops are due to resistances of series field, interpole winding, compensating winding and of armature respectively are in phase with armature current $I_a$. The reactance drops $I_aX_{se}$, $I_aX_i$, $I_aX_c$ and $I_aX_a$ are due to reactance of series field, interpole winding, compensating winding and of armature respectively lead current $I_a$ by $90^0$. The generated armature counter emf is $E_g$. The terminal phase voltage $V_p$ is equal to the phasor sum of $E_g$ and all the impedance drops in series.

$$V_p = E_g + I_aZ_{se} + I_aZ_i + I_aZ_c + I_aZ_a$$

The power factor angle between $V_p$ and $I_a$ is .

**4.4.3 Applications**

There are numerous applications where single-phase ac series motors are used, such as hair dryers, grinders, table-fans, blowers, polishers, kitchen appliances etc. They are also used for many other purposes where speed control and high values of speed are necessary.
4.5 Schrage Motor

Schrage motor is basically an inverted polyphase induction motor, with primary winding on the rotor and secondary winding on the stator. The primary winding on the rotor is fed through three slip rings and brushes at line frequency; secondary winding on the stator has slip frequency voltages induced in it.

The speed and power factor of slip ring induction motor can be controlled by injecting slip frequency voltage in the rotor circuit. If resultant rotor voltage increases, current increases, torque increases and speed increases. Depending on the phase angle of injected voltage, power factor can be improved. In 1911, K. H. Schrage of Sweden combined elegantly a SRIM (WRIM) and a frequency converter into a single unit.

4.5.1 Construction and Operation

Schrage motor has three windings- Two in Rotor and One in Stator.

*Primary winding:* Placed on the lower part of the slots of the Rotor. Three phase supply at line frequency is fed through slip rings and brushes which generates working flux in the machine.
Regulating winding: Placed on the upper part of the slots of the Rotor. These are connected to commutator segments in a manner similar to that of D.C. machine. Regulating windings are also known as tertiary winding / auxiliary winding / commutator winding.

Secondary winding: Same is phase wound & located on stator. Each winding is connected to a pair of brushes arranged on the commutator. Brushes are mounted on brush rockers. These are designed to move in opposite directions, relative to the centre line of its stator phase.

Brushes $A_1$, $B_1$ & $C_1$ move together and are $120^0$ apart.

Brushes $A_2$, $B_2$ & $C_2$ also move together and are $120^0$ apart.

Now the primary energized with line frequency voltage. Transformer action occurs between primary and regulating winding. Induction motor action occurs between primary and secondary windings. Commutator acting as a frequency converter converts line frequency voltage of regulating winding to slip frequency voltage and feeds the same to secondary winding on the stator.

Voltage across the brush pairs $A_1$ - $A_2$, $B_1$ - $B_2$ & $C_1$ - $C_2$ increases as brushes are separated.

Magnitude of voltage injected into the secondary winding depends on the angle of separation ‘$\theta$’ of the brushes $A_1$ & $A_2$, $B_1$ & $B_2$, $C_1$ & $C_2$. (‘$\theta$’ – Brush separation angle).

When primary is energized synchronously rotating field in clockwise direction is set up in the rotor core. Assume that the brushes are short circuited through commutator segment i.e. the secondary is short circuited. Rotor still at rest, the rotating field cuts the stationary secondary winding, induces an e.m.f. The stator current produce its own field. This stator field reacts with the rotor field thus a clockwise torque produced in the stator. Since the stator cannot rotate, as a reaction, it makes the rotor rotate in the counter clockwise direction.

Suppose that the rotor speed is $N_r$ rpm. Rotor flux is rotating with $N_S$ relative to primary & regulating winding. Thus the rotor flux will rotate at slip speed $(N_S - N_r)$ relative to secondary winding in stator with reference to space.
4.5.2 Speed Control

Speed of Schrage Motor can be obtained above and below Synchronous speed by changing the Brush position i.e. changing “θ” (‘θ’ – Brush separation angle).

In Fig: 4.22 (a) Brush pair on the same commutator segment i.e. the secondary winding short circuited. Thus the Injected voltage $E_j = 0$ and the machine operates as an Inverted Induction Motor so here $N_r < N_s$.

In Fig: 4.22 (b) Brushes parted in one direction which produces sub-synchronous speed. Injected voltage $E_j$, is obtained from the section of the regulating winding between them. If the centre line of this group of conductors is coincident with the centre line of the corresponding secondary phase, then $E_2$ and $E_j$ are in phase opposition. Neglecting impedance drop, $sE_2$ must be equal and opposite of $E_j$.

“β” is the angle between $E_2$ and $E_j$. $\beta=180^0$ and so here also $N_r < N_s$.

In Fig: 4.22 (c) Brushes parted in opposite direction which produces super-synchronous speed. Here $E_j$ is reversed relative to $E_2$ i.e. $\beta=0^0$ & $sE_2$ must also be reversed. This is occurring only because ‘s’ becoming negative i.e. The speed is thus above synchronous speed so $N_r > N_s$.

The commutator provides maximum voltage when the brushes are separated by one pole pitch. i.e. ‘θ’ = 180⁰.
4.5.3 Power Factor Improvement

This can be obtained by changing the phase angle of the injected voltage into the secondary winding. In this case one set of brushes is advanced more rapidly than the other set. Now the two centre lines do not coincide, have an angle ‘ρ’ between them. (“ρ” – Brush shift angle).

In Fig: 4.22 (d) Brush set is moved against the direction of rotation of rotor. In this case Speed decreases and the p.f. is improved.

![Fig: 4.22 (d) & (e)](image)

In Fig: 4.22 (e) Brush set is moved in the same direction of rotation of rotor. In this case Speed increases, the p.f. is also improved.

Both p.f. and speed can be controlled by varying ‘θ’ & ‘ρ’.

Thus ‘Ej Cos ρ’ and ‘Ej Sin ρ’ effect the speed and p.f. respectively. Fig: 4.23 show Variation of no load speed with Brush Separation.
4.5.4 Speed Torque Characteristics

Above discussion reveals that the Schrage Motor is almost a constant speed motor i.e. it has D.C Shunt motor characteristics. Figure 4.23 shows the typical speed-torque characteristics of Schrage motor.

4.5.5 Advantages & Shortcomings

*Advantages:*

(i) Good Speed Regulation.
(ii) High p.f. for high speed setting.
High efficiency at all speeds except \( N_s \)

**Shortcomings:**

(i) Operating voltage has to be limited to 700V because the power is to be supplied through slip rings.
(ii) Low p.f. at low speed settings.
(iii) Poor commutation.
(iv) High Cost.

**4.5.6 Applications**

Can be applied to any individual drive requiring variable speed, especially in knitting & Ring spinning applications, Cranes & Hoists Fans & Centrifugal Pumps, printing Machinery Conveyors, Packing machinery & Paper Mills etc.

**4.6 Universal Motors**

It is also commutator type motor. A universal motor is one which operates both on AC and DC supplies. It develops more horsepower per Kg. weight than any other AC motor mainly due to its high speed.

The principle of operation is the same as that of a DC motor. Though a universal motor resembles a DC series motor, it required suitable modification in the construction, winding and brush grade to achieve sparkles commutation and reduced heating when operated on AC supply, due to increased inductance and armature reaction.

A universal motor could therefore be defined as a series or a compensated series motor [Fig: 4.24 & Fig: 4.25 (a), (b)]designed to operate at approximately the same speed and output at either direct current or single phase alternating current of a frequency not greater than 50Hz, and of approximately the same RMS voltage. Universal motor is also named as AC single phase series motor.

![Fig: 4.24](image-url)
The main parts of a universal motor are an armature, field winding, stator stampings, frame and plates and brushed. The increased sparking at the brush position in AC operation is reduced by the following means:

- Providing commutating inter poles in the stator and connecting the interpole winding in series with the armature winding.
- Providing high contact resistance brushed to reduce sparking at brush positions.

### 4.6.1 Operation

A universal motor works on the same principles as a DC motor i.e. force is created on the armature conductors due to the interaction between the main field flux and the flux created by the current carrying armature conductors. A universal motor develops unidirectional torque regardless of whether it operated on AC or DC supply.

Fig: 4.25 (a),(b) & Fig: 4.26 shows the operation of a universal motor on AC supply. In AC operation, both field and armature currents change their polarities, at the same time resulting in unidirectional torque.
4.6.2 Characteristic

The speed of a universal motor inversely proportional to the load i.e. speed is low at full load and high, on no load.
The speed reaches a dangerously high value due to low field flux at no loads in fact the no load speed is limited only by its own friction and windage losses. As such these motors are connected with permanent loads or gear trains to avoid running at no load thereby avoiding high speeds.

Fig: 4.27 shows the typical torque-speed relation of a universal motor, both for AC and DC operations. This motor develops about 450 % of full load torque at starting, as such higher than any other type of single phase motor.

### 4.6.3 Applications

There are numerous applications where universal motors are used, such as hand drills, hair dryers, grinders, blowers, polishers, and kitchen appliances etc. They are also used for many other purposes where speed control and high values of speed are necessary like in vacuum cleaners, food mixers, portable drills and domestic sewage machines. Universal motors of a given horse power rating are significantly smaller than other kinds of a.c. motors operating at the same frequency.
Acknowledgement

The committee members gratefully acknowledge Google, scribd, NPTEL, openoffice, sumatra pdf, scilab for myriad suggestions and help for preparing this lecture note. The committee members also want to express their gratitude to the persons out there who think knowledge should be free and be accessible and sharable without any restrictions so that every single person on this planet has the same opportunity to explore, expand and become enlightened by the collective gifts of humankind.

However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books and above all confer with the concern faculty for thorough knowledge of this authoritative subject of electrical engineering.

**Text / Reference Books**

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*Wish you all the best*