EE705(B)
HIGH VOLTAGE D.C. TRANSMISSION

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Note


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Syllabus

Module I


Thyristor valve - valve firing - valve design consideration - Grading and damper circuit design - valve protection. Valve tests - Dielectrical and operational tests.

Module II


Module III


Module IV

Smoothing reactors - DC lines - DC line insulators - DC breakers - basic concept, characteristics, types and applications. Sources of reactive power - static VAR systems - Thyristor controlled reactor - Types of AC filters (Basic concept only) - DC filters - Carrier frequency and RI noise. Multiterminal DC system - Potential. Application and type. Modeling of DC network.

Simulation of HVDC system - system simulation - philosophy and tools only.
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Part I
Notes
Chapter 1

Introduction

1.1 Types of HVDC links

1. Monopolar link
   The monopolar link had got one conductor, usually of negative polarity, and ground or sea return.

2. Bipolar link
   The bipolar link has two conductors, one positive and the other negative. Each terminal has two converters of equal rated voltages in series on the dc side. The neutral points (junctions between converters) are grounded at one or both ends. If both neutrals are grounded, the two poles can also work independently. Normally they operate at equal current; then there is no ground current. In the event of a fault on one conductor, the other conductor with ground return can carry up to half of the rated load.

3. Homopolar link
   The homopolar link has two or more conductors all having the same polarity, usually negative, and always operates with ground return. In the event of a fault on one conductor, the entire converter is available for connection to the remaining conductor/s, which, having some overload capability can carry more than half of the rated power, at the expense of increased line loss. However this is not possible in a bipolar system due to the use of graded insulation for negative and positive poles. In this respect, a homopolar line is preferable where continuous ground current not deemed objectionable. An additional minor advantage is the low power loss due to corona. Negative polarity is preferred on overhead lines because of its smaller radio interference.
1.2 HVDC Transmission system

Essential components of HVDC systems are:

1. 6/12 pulse converters
2. Converter transformer with suitable ratio and tap changing
3. Filters both on the ac side and on the dc side to reduce the harmonics generated. There are two types of filters - tuned (on ac side) and band pass
4. A smoothing reactor to smoothen the DC current and other possible transient over-currents.
5. Shunt capacitor to complement the reactive power generated by the converters.
6. DC transmission line/cable

1.3 Comparison of HVDC and HVAC transmission systems

1.3.1 Economic Aspects

1. For a given power level, dc line requires less Right-of-Way (RoW), simpler and cheaper towers and reduced conductor and insulator costs. The power loss is also reduced with dc as there are only two conductors. The absence of skin effect is with dc is also beneficial in reducing power losses marginally. The dielectric loss in case of power cabled is also very less in case of dc transmission.

Let us assume that an ac line and a dc line using the same conductors and insulators are built. Assume that in each case, the current is limited by temperature rise. Then the dc current equals the rms current.

Assume also that the insulators withstand the same crest voltage to ground in each case. Then the direct voltage is $\sqrt{2}$ times the rms alternating voltage.

The dc power per conductor is

$$p_d = V_d I_d$$  \hspace{1cm} (1.1)

and the ac power per conductor is

$$p_a = V_a I_a \cos \phi$$  \hspace{1cm} (1.2)
where \( I_d \) and \( I_a \) are the currents per conductor, \( V_d \) and \( V_a \) the conductor-to-conductor ground voltages, and \( \cos \phi \) the power factor. The ratio is

\[
\frac{P_d}{P_a} = \frac{V_d I_d}{V_a I_a \cos \phi}
\]

(1.3)

Take \( \cos \phi = 0.945 \) and \( \frac{p_d}{p_a} = 1.5 \). Now compare a three phase, three conductor ac line with a bipolar two conductor dc line. The power capabilities of the respective circuits are

\[
P_d = 2p_d \\
P_a = 3p_a
\]

and the ratio is

\[
\frac{P_d}{P_a} = \frac{2p_d}{3p_a} = 1
\]

Both lines carry the same power. The dc line is simpler and cheaper, having two conductors instead of three.

2. The corona loss on dc conductors tend to be less significant than for ac and this also leads to the choice of economic size of conductors with dc transmission. The other factors that influence the line costs are the costs of compensation and terminal equipment. DC lines do not require compensation but the terminal equipment costs are increased due to the presence of converters and filters.

3. Length of Transmission line: The variation of cost of ac and dc transmission is shown in figure 1.2. DC transmission is economical only beyond a certain distance \( d^* \), called the breakeven distance. The breakeven distance also varies with the power transmitted.

![Figure 1.2: Variation of cost of ac and dc transmission vs. distance](image)

1.3.2 Technical Performance

1. **Current Limit**: The temperature of a conductor must be limited in order to avoid damage to the conductor itself (permanently increased sag) or, in case of a cable, to the insulation in contact with it. Hence the current in the conductor must be limited in accordance with its duration and the ambient temperature. The ac resistance of a conductor is higher than its dc resistance because of skin effect. For a given conductor, the current carrying capacity of HVDC system is more than the HVAC system for the same voltage level.

2. **Reactive power and voltage regulation (or voltage control)**: On long EHVAC overhead lines and on much shorter ac cables, the production and consumption of reactive power by the line itself constitutes a serious problem. On a line having series inductance \( L \) and...
shunt capacitance $C$ per unit length and operating voltage $V$ and current $I$, the line produces reactive power

$$Q_C = \omega CV^2$$

(1.4)

and consumes reactive power

$$Q_L = \omega LI^2$$

(1.5)

per unit length. The reactive power produced by the line equals that consumed by it, with no net production or consumption, if

$$\omega CV^2 = \omega LI^2$$

(1.6)

hence if

$$V/I = \left[\frac{L}{C}\right]^{1/2} = Z_s$$

(1.7)

In this case, the load impedance has the value $Z_s$, known as the surge impedance of the line. The power carried by the line so loaded is

$$P_n = VI = \frac{V^2}{Z_s}$$

(1.8)

and is called the surge impedance loading (SIL) or natural load. It is independent of distance and depends mainly on the voltage.

![Figure 1.3: Variation of voltage along the transmission line (uncompensated)](image)

For stipulated constant voltages at the sending end and receiving end, the voltage profile varies with line loading. The voltage profile is relatively flat only for a fixed level of power transfer corresponding to SIL ($P = P_n$). Under light loads ($P < P_n$), the mid-point voltage may exceed the upper permissible voltage limit and at high loads ($P > P_n$), the voltage may dip below the lower permissible limit.

Another limitation of long lines is the high voltage at an open end (the Ferranti effect). This is important when a line is being put into service by first connecting one end of it to the main ac system, for it is not feasible to close both ends at exactly the same moment.

The maintenance of constant voltages at the two ends requires reactive power control from inductive to capacitive (mid-point shunt, inductive or capacitive or static Var compensators, compensation) as the line loading is increased. The reactive power requirements also increase with increase in line lengths. Although DC converter stations with line commutated converters require reactive power (drawn from the ac side, usually shunt capacitors are provided for this) related to the line loadings (independent of line length), the line itself does not require reactive power, and the voltage drop on the line itself is merely the resistive drop. For distances more than 400km the total reactive power requirement of HVDC system is less than that of HVAC system, for the same power to be transmitted.
In the case of cables (submarine or underground), they are always operated at a load much below the SIL in order to avoid over-heating. Consequently, the reactive power produced by charging the shunt capacitance greatly exceeds that consumed by the series inductance. In a 50Hz power cable of length approximately 40km, the charging current alone equals the rated current. Shunt compensation theoretically would solve this problem, but practically it is very difficult to lay and maintain (or repair) such cables. DC cables have no such limitation.

3. **Stability**: The stability of an ac system is its ability to operate with all synchronous machines in synchronism. If a long ac line is loaded to a certain value, known as its steady-state stability limit, the synchronous machines at the sending end will go out of synchronism with those at the receiving end. This will in turn give rise to objectionable fluctuations in voltage.

Even if a line is operated below its steady-state limit, the machines at the sending and receiving ends may lose synchronism after some large disturbance, notably a short circuit, unless the line is operated below its transient stability limit, which is always lower than the steady-state limit. Practically speaking, the steady-state stability limit is the transient stability limit for a very small disturbance.

The problem of stability or synchronous operation constitutes the most serious limitation of a long ac transmission system. The power transmitted from the sending end to the receiving end in a lossless ac power system is given by

\[ P = \frac{E_1 E_2}{X} \sin \delta \]  

(1.9)

where \( E_1 \) and \( E_2 \) are the voltages of sending end and receiving end of the power system, \( \delta \) is the phase difference between these voltages (power angle or internal angle), and \( X \) is the reactance of the connecting network. Thus the maximum power that can be transferred over a long transmission line is inversely proportional to the line reactance. Whereas in DC transmission, the power transmitted is limited only by the current carrying capacity of the conductors. A dc transmission link in itself has no stability problem.

![Figure 1.4: Power transfer capability vs. distance](image)

4. **Circuit Breakers**: AC circuit breakers take advantage of the current zeros that occur twice per cycle. They are designed to increase the breakdown strength of the arc path between contacts so rapidly that the arc does not re-strike. DC circuit breakers do not have this natural advantage and therefore have to force the current to zero. In simple two-terminal dc transmission, the lack of dc circuit breakers have not been felt, because faults on the dc line or in the converter are cleared by using the control grids of the converter valves to block the current temporarily.

5. **Ground Return**: A two conductor bipolar dc line is more reliable than a three conductor ac line, because, in the event of a fault on one conductor, the other conductor can continue...
to operate with ground return during the period required for operating the fault. The operation of an ac line with ground return is not feasible on account of high impedance of such a circuit and the telephonic interference caused by such an operation. The ground return in dc transmission is objectionable only when buried metallic structures (such as pipes) are present and are subject to corrosion with DC current flow.

6. **Problems of ac interconnection:** When two power systems are connected through ac ties (synchronous interconnection), the automatic generation control of both systems has to be coordinated using tie-line power and frequency signals. Even with coordinated control, of interconnected systems, the operation of ac ties can be problematic due to:

   (a) presence of large power oscillations which can lead to frequent tripping
   (b) increase in fault levels
   (c) transmission of disturbances from one system to the other.

The controllability of power flow in dc lines eliminates all the above problems. In addition, for asynchronous dc ties, there is no need of coordinated control. It is obvious that two systems which have different nominal frequencies cannot be interconnected directly with ac ties and require the use of dc ties.

### 1.3.3 Reliability

There are two measures of overall system reliability.

1. **Energy Availability:** Is defined as

   \[
   \text{Energy Availability} = 100 \times \left( 1 - \frac{\text{equivalent outage time}}{\text{total time}} \right) \%
   \]

   where equivalent outage time is the product of the actual outage time and the fraction of system capacity lost due to outage.

2. **Transient Reliability:** This is a factor specifying the performance of HVDC systems during recordable faults on the associated AC systems.

   \[
   \text{Transient Reliability} = \frac{\text{No. of times HVDC system performed as designed}}{\text{no. recordable ac faults}}
   \]

   Recordable ac system faults are those faults which cause one or more AC bus phase voltage to drop below 90

   A bipolar dc line is as reliable as a double circuit ac line with the same power transfer capability. This is because of the fact that failure of one pole does not affect the operation of the other pole (with ground return).

### 1.4 Applications of HVDC transmission system

1. Long distance bulk power transmission
2. Power transmission using underground or underwater cables (for distance > 32km)
3. Asynchronous interconnection of AC systems operating at different frequencies or where independent control of systems is desired
4. Control and stabilisation of power flows in AC ties in an integrated power system
Chapter 2

HVDC Converters

2.1 Choice of converter configuration

The choice of converter configuration is made on the following requirements:

1. High pulse number
2. Valve utilization factor, $\frac{PIV}{V_{d0}}$, should be as low as possible
3. $\frac{V_{d0}}{E}$ should be as high as possible
4. Transformer Utilization Factor (TUF) should be as low as possible

2.1.1 Pulse number

The number of pulsations or ripples of dc voltage per cycle of ac voltage is known as pulse number of a converter.

If $q$ is the number of valves in a commutation group$^1$ and $r$ of these are connected in parallel and $s$ of them are connected in series, the pulse number is given by

$$p = qrs$$  \hspace{1cm} (2.1)

High pulse numbers will result in lesser harmonics generated as the characteristic harmonics generated on the ac side is given by $h = np \pm 1$ and on the dc side is given by $h = np$, where $n$ is an integer. Thus low order harmonics, which have comparatively higher magnitudes are eliminated when $p$ is high.

Higher the pulse number, higher will be the average output dc voltage level per cycle of ac wave.

Since it is easy to design filters (low pass filters) that eliminate higher order harmonics, high pulse number is preferred.

Higher pulse number also results in better utilisation of converter transformer and valves.

The pulse number cannot be increased indiscriminately as the number of valves in the circuit increases for increase in pulse number. This will increase the overall cost of the converter station.

2.1.2 Valve utilization factor

The valve utilization factor (VUF) is given by

$$VUF = \frac{PIV}{V_{d0}}$$  \hspace{1cm} (2.2)

$^1$A group of valves in which only one valve conducts at any instant, neglecting overlap.
where PIV is the peak inverse voltage, the maximum reverse bias voltage that the valve is subjected to and $V_{d0}$ is average output dc voltage when there is no delay in firing the valves i.e. $\alpha = 0^\circ$. It is always good when PIV is less. This is one reason why we go for bridge converter circuits. The converter configuration should be so chosen such that $V_{d0}$ is maximum.

If there are $q$ valves in a commutation group, each valve will conduct for a period of $2\pi/q$. The average maximum dc output voltage of the converter is

$$V_{d0} = \frac{1}{2\pi/q} \int_{-\pi/q}^{\pi/q} E_m \cos \omega t \, d\omega$$

$$= \frac{q}{\pi} E_m \sin \frac{\pi}{q}$$

(2.3)

(2.4)

where $E_m = \sqrt{2}E$, is the peak value of the input alternating voltage and $\omega = 2\pi f$ is the supply frequency in rad/sec. If there are $s$ number of series connections, then the dc voltage is

$$V_{d0} = \frac{sq}{\pi} E_m \sin \frac{\pi}{q}$$

(2.5)

The expression for PIV is

$$PIV = \begin{cases} 2E_m & \text{if } q \text{ is even} \\ 2E_m \cos \frac{\pi}{2q} & \text{if } q \text{ is odd} \end{cases}$$

(2.6)

and therefore,

$$\frac{PIV}{V_{d0}} = \begin{cases} \frac{2\pi}{sq \sin \frac{\pi}{q}} & \text{if } q \text{ is even} \\ \frac{2\pi}{sq \cos \frac{\pi}{2q}} & \text{if } q \text{ is odd} \end{cases}$$

(2.7)

### 2.1.3 $\frac{V_{d0}}{E}$

This ratio should be high as we prefer a very minimal voltage drop in the conversion circuit. Using Eq. (2.5), we get

$$\frac{V_{d0}}{E} = \frac{\sqrt{2qs}}{\pi} \sin \frac{\pi}{q}$$

(2.8)

### 2.1.4 TUF

TUF is defined as the ratio of transformer rating (of the valve side) to the dc power output. This value should be as low as possible. The current rating of the transformer is given by

$$I_t = \frac{I_d}{r \sqrt{2}}$$

(2.9)

where $I_d$ is the dc current through the link. The transformer rating will be

$$P_t = \frac{E_m}{\sqrt{2}} I_t = \frac{E_m I_d}{r \sqrt{2q}}$$

(2.10)

Thus TUF can be calculated as

$$TUF = \frac{P_t}{V_{d0} I_d} = \frac{\pi}{\sqrt{2q \sin \frac{\pi}{q}}}$$

(2.11)

An important point to be noted here is that the value of TUF is dependant on the value $q$ only.
From the table it can be seen that sl. no. 1 and 3 are suitable, but TUF is better for sl. no. 3. Since 3 phase supply is used, the ease of connection is an added advantage in using sl. no. 3.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>q</th>
<th>r</th>
<th>s</th>
<th>$\frac{PIV}{V_{d0}}$</th>
<th>$\frac{V_{d0}}{E}$</th>
<th>TUF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1.047</td>
<td>2.700</td>
<td>1.571</td>
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<td>2</td>
<td>3</td>
<td>1</td>
<td>3.142</td>
<td>0.900</td>
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<tr>
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<td>2</td>
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<td>2.340</td>
<td>1.481</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2.094</td>
<td>1.169</td>
<td>1.481</td>
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<td>1</td>
<td>2.094</td>
<td>1.350</td>
<td>1.814</td>
</tr>
</tbody>
</table>

Table 2.1: Converter Configuration - 6 pulse
Chapter 3

Analysis of Converters - Graetz Circuit

3.1 Assumptions made for analysis

The following assumptions are made to make analysis simpler.

1. The ac power source is a balanced sinusoidal source with constant amplitude and frequency and has no impedance.
2. The dc current is ripple free i.e. the dc load has infinite inductance.
3. Valves have zero forward resistance when conducting and infinite resistance when not conducting.
4. Valves are ignited at equal intervals of 60°.

3.2 Modes of operation

Depending on the number of valves conducting at any instant, there are different modes of operation. Ideally, only two valves conducts (one from upper commutation group and one from the lower commutation group) at any given time and it is assumed that the current transfer from outgoing valve to the incoming valve is instantaneous. This does not happen in real time circuits i.e. there will be more than two valves conducting simultaneously and this phenomenon is called overlap. The phenomenon of overlap is caused mainly due to

1. leakage reactance of the transformer, and
2. the unsymmetric firing of valves

When there is overlap, the current transfer is not instantaneous.

Depending on the period of overlap, denoted by $u$, there are different modes of operation. They are

Mode 1 $u = 0^\circ$, 2 valve conduction mode
When $u = 0^\circ$, there is no overlap. This is the ideal mode of working of a converter circuit.

Mode 2 $u < 60^\circ$, 2 or 3 valve conduction mode
This is the most practical mode of working of a converter circuit.

Mode 3 $u = 60^\circ$, 3 valve conduction mode

Mode 4 $u > 60^\circ$, 3 or 4 valve conduction mode
3.3 Preliminary data for analysis

Before we move on to the analysis, we need to define the voltage phasors and the initial condition of the circuit.

Taking $e_{ba}$ as the reference voltage, the other voltages can be derived as

$$e_{ba} = \sqrt{3}E_m \sin \omega t$$  \hspace{1cm} (3.1)

$$e_a = E_m \sin \left( \omega t + \frac{5\pi}{6} \right)$$  \hspace{1cm} (3.2)

$$e_b = E_m \sin \left( \omega t + \frac{\pi}{6} \right)$$  \hspace{1cm} (3.3)

$$e_c = E_m \sin \left( \omega t - \frac{\pi}{2} \right)$$  \hspace{1cm} (3.4)

$$e_{cb} = \sqrt{3}E_m \sin \left( \omega t - \frac{2\pi}{3} \right)$$  \hspace{1cm} (3.5)

$$e_{ac} = \sqrt{3}E_m \sin \left( \omega t + \frac{2\pi}{3} \right)$$  \hspace{1cm} (3.6)

$$e_{bc} = \sqrt{3}E_m \sin \left( \omega t + \frac{\pi}{3} \right)$$  \hspace{1cm} (3.7)

A valve can be triggered when its anode to cathode voltage is positive. It can be delayed by angle $\alpha$.

3.4 Mode 1: Without overlap

In this mode of operation $u = 0^\circ$ and only two valves conduct at any instant and the current transfer is instantaneous.

3.4.1 DC output voltage

Since each pair of valves conducts for a period of $60^\circ$, the voltage ripples will be symmetric over each period. Thus the average dc voltage can be calculated as

$$V_d = \frac{1}{\pi/3} \int_{\alpha}^{\pi/3+\alpha} e_{bc} \, d\omega t$$

$$= \frac{3\sqrt{3}E_m}{\pi} \int_{\alpha}^{\pi/3+\alpha} \sin \left( \omega t + \frac{\pi}{3} \right) \, d\omega t$$

$$= \frac{3\sqrt{3}E_m}{\pi} \cos \alpha$$

$$= V_{d0} \cos \alpha$$

(3.11)

where $V_{d0}$ is called the ideal no-load direct voltage and is defined as

$$V_{d0} = \frac{3\sqrt{3}E_m}{\pi}$$  \hspace{1cm} (3.12)

The polarity of $V_d$ depends upon the value of $\alpha$. Thus $V_d$ varies from $V_{d0}$ to $-V_{d0}$, which implies that the same converter can act as a rectifier or inverter depending upon the value of $\alpha$. The dc output voltage is maximum when $\alpha = 0^\circ$ and zero when $\alpha = 90^\circ$.

3.4.2 Power factor

With losses in the converter neglected, the ac power must be equal to the dc power, i.e.

$$3E_{LN}I_{L1} \cos \phi = V_dI_d = V_{d0}I_d \cos \alpha$$

(3.13)
where $I_{L1}$ is the rms value of the fundamental component of the alternating line current. The line current has the waveform as shown in figure and consists of positive and negative rectangular pulses of magnitude of $I_d$ and width $2\pi/3$. This shape is independent on $\alpha$ as long as there is no overlap. The rms value of the fundamental component can be found out using Fourier analysis

$$I_{L1} = \frac{1}{\sqrt{2}} \left[ \frac{2}{\pi} \int_{-\pi/3}^{+\pi/3} I_d \cos \theta \, d\theta \right]$$

$$= \frac{\sqrt{3}}{\pi} I_d \left[ \sin \frac{\pi}{3} \right]$$

$$= \frac{\sqrt{6}}{\pi} I_d$$

Substituting Eq.(3.16) and Eq.(3.12) into Eq.(3.13), we get

$$\cos \phi = \cos \alpha$$

The above equations shows that power factor decreases when firing angle is increased and thus the converter requires more reactive power.

### 3.4.3 Transformer current on the secondary side

The rms value of the overall current is given by

$$I = \left[ \frac{1}{2\pi} \int_0^{2\pi} I_d^2 \, d\omega t \right]^{1/2}$$

$$= \frac{I_d}{2\pi} \left[ \int_{\pi/6}^{5\pi/6} 1 \, d\omega t + \int_{7\pi/6}^{11\pi/6} 1 \, d\omega t \right]^{1/2}$$

$$= \sqrt{\frac{2}{3}} I_d$$

The rms value of the $h^{th}$ harmonic current $I_h$ is given by

$$I_h = \frac{I_{1rns}}{h}$$

where $I_{1rns}$ is the rms value of the fundamental component, $h$ is the order of harmonics given by

$$h = np \pm 1$$

where $p$ is the number of pulses, $n$ is an integer.

### 3.4.4 Transformer rating

Transformer rating (denoted as $T$) in Volt-Amperes can be calculated using

$$T = \frac{\pi}{3} V_{ab} I_d$$

### 3.4.5 Power rating of the valve

### 3.5 With overlap

Due to the inductance of the transformer, the current in it can vary only at a finite rate and thus the transfer of current from one phase to another phase will take finite time, called the commutation time or overlap time.
3.5.1 2 or 3 valve conduction mode

DC Voltage

When valves 1 and 3 are conducting (i.e. during the period of overlap), lines a and b are short circuited.

\[ V_a = e_a - L \frac{dI_a}{dt} \]  \hspace{1cm} (3.23)
\[ V_a = e_b - L \frac{dI_b}{dt} \]  \hspace{1cm} (3.24)

Adding the above two equations

\[ 2V_a = e_a + e_b - L \frac{d(I_a + I_b)}{dt} \]  \hspace{1cm} (3.25)

But \( I_a + I_b = I_d \), the dc current and is assumed to be a constant. Therefore

\[ V_a = e_a + e_b - L \frac{dI_d}{dt} \]  \hspace{1cm} (3.26)
\[ \therefore V_a = \frac{e_a + e_b}{2} \]  \hspace{1cm} (3.27)

For a balanced 3 phase supply \( e_a + e_b + e_c = 0 \), and hence

\[ V_a = \frac{e_a + e_b}{2} = \frac{e_c}{2} \]  \hspace{1cm} (3.28)

The average dc terminal voltage can be obtained as

\[ V_d = V_a - e_c = \left\{ \begin{array}{ll}
\frac{e_a + e_b}{2} - \frac{e_c}{2} = -\frac{3e_c}{2} & \alpha \leq \omega t \leq u + \alpha \\
e_b - e_c = e_{bc} & u + \alpha \leq \omega t \leq \frac{\pi}{3} + \alpha
\end{array} \right. \]  \hspace{1cm} (3.29)

Therefore,

\[ V_d = \frac{3}{\pi} \left[ \int_{\alpha}^{u+\alpha} e_{bc} \, d\omega t + \int_{u+\alpha}^{\pi/3+\alpha} e_{bc} \, d\omega t \right] \]
\[ = \frac{3\sqrt{3}E_m}{2\pi} \left[ \cos \alpha + \cos (\alpha + u) \right] \]
\[ = \frac{V_{d0}}{2} \left[ \cos \alpha + \cos (\alpha + u) \right] \]  \hspace{1cm} (3.30)
DC Current

From Eq. (3.23) and (3.24),
\[
e_a - L \frac{dI_a}{dt} = e_b - L \frac{dI_b}{dt}
\]
\[
L \left[ \frac{dI_b}{dt} - \frac{dI_a}{dt} \right] = e_b - e_a
\]  
(3.31)
(3.32)

Since \(I_a = I_d - I_b\),
\[
\frac{dI_a}{dt} = - \frac{dI_b}{dt}
\]
(3.33)

Therefore Eq.(3.32) becomes
\[
2L \frac{dI_b}{dt} = e_{ba} = \sqrt{3}E_m \sin (\omega t)
\]
(3.34)

Integrating on both sides
\[
I_b = -\frac{\sqrt{3}E_m}{2\omega L} \cos (\omega t) + A
\]
(3.35)

where \(A\) is the integration constant. \(A\) can be calculated from the initial conditions. At \(\omega t = \alpha\), the current through valve 3 is zero i.e. \(I_3 = 0\).

\[
\therefore A = \frac{\sqrt{3}E_m}{2\omega L} \cos \alpha
\]
(3.36)

Eq.(3.35) now becomes
\[
I_b = -\frac{\sqrt{3}E_m}{2\omega L} \cos (\omega t) + \frac{\sqrt{3}E_m}{2\omega L} \cos \alpha
\]
(3.37)
\[
= \frac{\sqrt{3}E_m}{2\omega L} \left[ \cos \alpha - \cos \omega t \right]
\]
(3.38)

Eq.(3.38) is valid only during the commutation period i.e. from \(\alpha\) to \(\alpha + \pi\). The current of the incoming valve during commutation, consists of a constant (dc) term and a sinusoidal term. The sinusoidal term lags behind the commutating voltage by 90°, as it should in a purely inductive circuit. The equation for \(I_a\) can be deduced from \(I_a = I_d - I_b\). At the end of commutation i.e. at \(\omega t = \alpha + \pi\), \(I_b = I_d\). Therefore
\[
I_d = \frac{\sqrt{3}E_m}{2\omega L} [\cos \alpha - \cos (\alpha + \pi)]
\]
(3.39)

Let \(I_s = \frac{\sqrt{3}E_m}{2\omega L}\), therefore
\[
I_d = I_s [\cos \alpha - \cos (\alpha + \pi)]
\]
(3.40)

Power Factor

\[
3E_{LN} I_{L1} \cos \phi = V_d I_d
\]
(3.41)

From eq.(3.30),
\[
V_d = \frac{3\sqrt{3}E_{LN}}{2\pi} [\cos \alpha + \cos (\alpha + \pi)]
\]
(3.42)
Therefore,

\[ 3E_{LN} I_L \cos \phi = \frac{3\sqrt{6}E_{LN}}{2\pi} \left[ \cos \alpha + \cos (\alpha + u) \right] I_d \]  

\[ I_L \cos \phi = \left[ \frac{\sqrt{6}}{\pi} I_d \right] \left[ \frac{\cos \alpha + \cos (\alpha + u)}{2} \right] \]  

(3.43)  

(3.44)

Using the following approximation,

\[ I_L \approx \frac{\sqrt{6}}{\pi} I_d \]  

\[ \cos \phi = \frac{\cos \alpha + \cos (\alpha + u)}{2} \]  

(3.45)  

(3.46)

### 3.5.2 Equivalent Circuit

From eq.(3.39),

\[ \cos (\alpha + u) = \cos \alpha - \frac{2\omega LI_d}{\sqrt{3}E_m} \]  

Substituting the above equation into eq.(3.30) and eliminating \( \cos (\alpha + u) \),

\[ V_d = \frac{3\sqrt{3}E_m}{2\pi} \left[ 2 \cos \alpha - \frac{2\omega LI_d}{\sqrt{3}E_m} \right] \]  

\[ = \frac{3\sqrt{3}E_m}{\pi} \cos \alpha - \frac{3\omega L}{\pi} I_d \]  

\[ = V_{d0} \cos \alpha - \frac{3\omega L}{\pi} I_d \]  

(3.47)  

(3.48)  

(3.49)

Let \( R_c = \frac{3\omega L}{\pi} \), be defined as the equivalent commutation resistance. Although it is called as resistance, it does not consume power and is used only for representing a voltage drop due to overlap. Therefore

\[ V_{dr} = V_{d0} \cos \alpha - R_c I_d \]  

(3.50)

Eq.(3.50) is valid for the rectifier and the same equation is modified so that it represents the inverter. Let \( \pi - \alpha = \beta \) and \( \pi - (\alpha + u) = \gamma \), where \( \gamma \) is called the extinction angle and \( \beta \) is called the angle of advance.

Eq.(3.30) can now be modified as

\[ V_{di} = -\frac{V_{d0}}{2} (\cos (\pi - \beta) + \cos (\pi - \gamma)) \]  

The voltage is indicated as negative, since the representation is for the inverter side. Therefore,

\[ V_{di} = \frac{V_{d0}}{2} (\cos \beta + \cos \gamma) \]  

(3.51)

From eq.(3.12),

\[ V_{di} = -V_{d0} \cos (\pi - \beta) + R_c I_d \]  

\[ V_{di} = V_{d0} \cos \beta + R_c I_d \]  

(3.52)

Modifying eq.(3.51), we get

\[ \cos \beta = -\cos \gamma + \frac{2V_{di}}{V_{d0}} \]  

(3.53)
Substituting eq.(3.53) into eq.(3.52)

\[
V_{di} = V_{ dni}\left[-\cos \gamma + \frac{2V_{di}}{V_{ dni}}\right] + R_c I_d \\
= -V_{ dni}\cos \gamma + 2V_{di} + R_c I_d \\
\therefore V_{di} = V_{ dni}\cos \gamma - R_c I_d
\]

(3.54)

3.5.3 3 or 4 valve conduction mode

Operation of bridge converter with overlap angle \( \alpha > 60^\circ \) is abnormal, being encountered only under overload, dc short circuit, or low alternating voltage. However, it is self curing, i.e. it will continue conducting in 3 valve mode after overlap period. When 4 valves are conducting, they constitute a 3 phase short circuit on the ac side and a pole-to-pole short circuit on the dc side. When 3 valves are conducting, they constitute a line-to-line short circuit on the ac side (as seen in the previous section).

In this mode of operation, when 4 valves are conducting, the direct voltage is zero, and when 3 valves are conducting, the output is an arc of a sine wave of amplitude \( \frac{1}{5}E_{m} \).

DC Voltage

The average dc terminal voltage can be obtained as

\[
V_d = \begin{cases} 
0 & \alpha \leq \omega t \leq \alpha + u - \frac{\pi}{3} \\
-1.5E_c & \alpha + u - \frac{\pi}{3} \leq \omega t \leq \alpha + \frac{\pi}{3}
\end{cases}
\]

(3.55)
Therefore the output dc voltage is

\[ V_d = \frac{3}{\pi} \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} -1.5e_c \]  

(3.56)

\[ = -\frac{9}{2\pi} \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} E_m \sin\left(\omega t - \frac{\pi}{2}\right) d\omega t \]

\[ = \frac{9}{2\pi} E_m \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} \cos \omega t d\omega t \]

\[ = \frac{9}{2\pi} E_m \left[ \sin \omega t \right]_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} \]

\[ = \frac{9}{2\pi} E_m \left[ \cos \left(\omega t - \frac{\pi}{2}\right) \right]_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} \]

\[ = \frac{9}{2\pi} E_m \left[ \cos \left(\alpha - \frac{\pi}{6}\right) - \cos \left(\alpha + u - \frac{5\pi}{6}\right) \right] \]

\[ = \frac{9}{2\pi} E_m \left[ \cos \left(\alpha - \frac{\pi}{6}\right) + \cos \left(\alpha + u + \frac{\pi}{6}\right) \right] \]

\[ = \frac{\sqrt{3}}{2} V_{eb0} \left[ \cos \left(\alpha - \frac{\pi}{6}\right) + \cos \left(\alpha + u + \frac{\pi}{6}\right) \right] \]

(3.57)

**DC Current**

When valves 6,1,2,3 are conducting,

\[ e_a - L \frac{dI_a}{dt} = e_b - L \frac{dI_b}{dt} \]  

(3.58)

\[ e_a - L \frac{dI_a}{dt} = e_c - L \frac{dI_c}{dt} \]  

(3.59)

Using the above two equations, we get

\[ e_a - e_b = L \frac{dI_a}{dt} - L \frac{dI_b}{dt} \]  

(3.60)

\[ e_a - e_c = L \frac{dI_a}{dt} - L \frac{dI_c}{dt} \]  

(3.61)

Adding eq.(3.60) and eq.(3.61)

\[ 2e_a - e_b - e_c = 2L \frac{dI_a}{dt} - L \frac{dI_b}{dt} - L \frac{dI_c}{dt} \]

\[ 3e_a = 2L \frac{dI_a}{dt} - L \frac{dI_b}{dt} - L \frac{dI_c}{dt} \]  

(\because e_a + e_b + e_c = 0)

Since, \( I_a + I_b = I_d = -I_c \), we get

\[ 3e_a = 3L \frac{dI_a}{dt} \]

\[ \therefore \frac{dI_a}{dt} = \frac{e_a}{L} \]  

(3.62)

Considering the entire system,

\[ \frac{dI_a}{dt} = \frac{e_a}{L} = \frac{dI_1}{dt} = \frac{-dI_3}{dt} = \frac{E_m}{L} \sin \left(\omega t + \frac{5\pi}{6}\right) \]  

(3.63)
Integrating on both sides,

\[ I_3 = \int_0^\omega \frac{E_m}{L} \sin \left( \omega t + \frac{5\pi}{6} \right) \, dt \]

\[ = \frac{E_m}{\omega L} \left[ \cos \left( \omega t + \frac{5\pi}{6} \right) - \cos \left( \alpha + \frac{5\pi}{6} \right) \right] \]

\[ = \frac{E_m}{\omega L} \left[ -\cos \left( \omega t - \frac{\pi}{6} \right) + \cos \left( \alpha - \frac{\pi}{6} \right) \right] \quad (3.64) \]

The valves 6, 1, 2 & 3 will conduct during the period \( \alpha \leq \omega t \leq \alpha + \pi - \frac{\pi}{3} \). Therefore at \( \omega t = \alpha + \pi - \frac{\pi}{3} \), the current through valve 3 will be

\[ I_3 = \frac{E_m}{\omega L} \left[ -\cos \left( \alpha + u - \frac{\pi}{2} \right) + \cos \left( \alpha - \frac{\pi}{6} \right) \right] \quad (3.65) \]

At \( \omega t = \alpha + u - \frac{\pi}{3} \), valve 6 will turn off completely and the valves 1, 2, & 3 will conduct (3 valve conduction). Current through valve 3 during this period can be found by integrating the current (see eq.(3.34)) during this period and adding it to the initial current (eq.(3.65)).

\[ I_3 = \int_{u + \alpha - \frac{\pi}{3}}^{\omega + \alpha - \frac{\pi}{3}} \frac{\sqrt{3}E_m}{2L} \sin \omega t \, dt + \frac{E_m}{\omega L} \left[ -\cos \left( \alpha + u - \frac{\pi}{2} \right) + \cos \left( \alpha - \frac{\pi}{6} \right) \right] \]

\[ = \frac{\sqrt{3}E_m}{2\omega L} \left[ -\cos \omega t + \cos \left( u + \alpha - \frac{\pi}{3} \right) \right] + \frac{E_m}{\omega L} \left[ -\cos \left( \alpha + u - \frac{\pi}{2} \right) + \cos \left( \alpha - \frac{\pi}{6} \right) \right] \]

Valves 1, 2 & 3 will conduct during the period \( \alpha + u + \frac{\pi}{3} \leq \omega t \leq \alpha + \frac{\pi}{3} \). Therefore, at \( \omega t = \alpha + \frac{\pi}{3} \)

\[ I_3 = \frac{\sqrt{3}E_m}{2\omega L} \left[ \cos \left( u + \alpha - \frac{\pi}{3} \right) - \cos \left( \alpha + \frac{\pi}{3} \right) \right] + \frac{E_m}{\omega L} \left[ -\cos \left( \alpha + u - \frac{\pi}{2} \right) + \cos \left( \alpha - \frac{\pi}{6} \right) \right] \]

\[ = \frac{E_m}{\omega L} \left[ \cos \left( \alpha - \frac{\pi}{6} \right) - \cos \left( \alpha + u - \frac{\pi}{2} \right) + \frac{\sqrt{3}}{2} \cos \left( \alpha + u - \frac{\pi}{3} \right) - \frac{\sqrt{3}}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right] \]

By vector addition,

\[ -\cos \left( \alpha + u - \frac{\pi}{2} \right) + \frac{\sqrt{3}}{2} \cos \left( \alpha + u - \frac{\pi}{3} \right) = \frac{1}{2} \cos \left( \alpha + u + \frac{\pi}{6} \right) \] \quad (3.66)

\[ \therefore I_3 = \frac{E_m}{\omega L} \left[ \cos \left( \alpha - \frac{\pi}{6} \right) + \frac{1}{2} \cos \left( \alpha + u + \frac{\pi}{6} \right) - \frac{\sqrt{3}}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right] \quad (3.67) \]

At \( \omega t = \alpha + \frac{\pi}{3} \), valve 4 is fired. Now, 4 valves (1, 2, 3 & 4) will conduct during the period \( \alpha + \frac{\pi}{3} \leq \omega t \leq \alpha + u \). Analysis is done in a similar way as was done when valves 6, 1, 2 & 3 were conducting.

\[ e_a - L \frac{dI_a}{dt} = e_b - L \frac{dI_b}{dt} = e_c - L \frac{dI_c}{dt} \]

Combining the above equations.

\[ 2e_b - e_a - e_c = 2L \frac{dI_b}{dt} - L \frac{dI_a}{dt} - L \frac{dI_c}{dt} \]

\[ \therefore e_b = L \frac{dI_b}{dt} = L \frac{dI_3}{dt} = E_m \sin \left( \omega t + \frac{\pi}{6} \right) \] \quad (3.68)
The equation for $I_3$ can now be written as

$$I_3 = \int_{\alpha + \frac{\pi}{6}}^{\omega t} \frac{E_m}{L} \sin (\omega t + \frac{\pi}{6}) \, d\omega t$$

$$+ \frac{E_m}{\omega L} \left[ \cos \left( \alpha - \frac{\pi}{6} \right) + \frac{1}{2} \cos \left( \alpha + u + \frac{\pi}{6} \right) - \frac{\sqrt{3}}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right]$$

$$= \frac{E_m}{\omega L} \left[ \cos \left( \alpha - \frac{\pi}{2} \right) - \cos \left( \omega t + \frac{\pi}{6} \right) \right]$$

$$+ \frac{E_m}{\omega L} \left[ \cos \left( \alpha - \frac{\pi}{6} \right) + \frac{1}{2} \cos \left( \alpha + u + \frac{\pi}{6} \right) - \frac{\sqrt{3}}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right]$$

At $\omega t = \alpha + u$

$$I_d = I_3$$

$$= \frac{E_m}{\omega L} \left[ \cos \left( \alpha + \frac{\pi}{2} \right) \cos \left( \alpha + u + \frac{\pi}{6} \right) \right]$$

$$+ \frac{E_m}{\omega L} \left| \cos \left( \alpha - \frac{\pi}{6} \right) + \frac{1}{2} \cos \left( \alpha + u + \frac{\pi}{6} \right) - \frac{\sqrt{3}}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right|$$

Finally,

$$I_d = \frac{E_m}{2\omega L} \left[ \cos \left( \alpha - \frac{\pi}{6} \right) - \cos \left( \alpha + u + \frac{\pi}{6} \right) \right] \quad (3.69)$$
Chapter 4

Control of HVDC Systems

4.1 Desired Control Features

1. The control system should not be sensitive to normal variation in voltage and frequency of the ac supply system.

2. Control should be fast, reliable and easy (simple) to implement.

3. It should have continuous operating range from full rectification to full inversion.

4. Control should be such that, it should require less reactive power (in order to have a good power factor).

5. Under steady state conditions, the values must be fired symmetrically.

6. The control should be able to limit the maximum current so as to avoid damage to valves and other current carrying devices. It should also limit the fluctuations of current due to fluctuation of alternating voltage.

7. Power can be controlled independently and smoothly which can be done by controlling the current and/or the voltage simultaneously in the link.

8. Control should be such that it can be used for protection of line and converter
   - prevention of commutation failure of inverter
   - prevention of arc back of the rectifier valves

4.2 Principles of DC link control

Consider the steady state equivalent circuit of a 2 terminal dc link shown in figure. This is based on the assumption that all the series connected bridges in both poles of a converter station are identical and have same delay angles. Also, the number of series connected bridges \( n_b \) in both stations (rectifier and inverter) are the same. Therefore the voltage source at the rectifier side can be expressed as

\[
V_{d,rect} = n_b V_{abr} \cos \alpha
\]  

(4.1)

and that of the inverter side as

\[
V_{d,inv} = n_b V_{abi} \cos \gamma
\]  

(4.2)
4.3 Control Characteristics

4.4 Equivalent Circuit

\[ I_d = \frac{V_{dq} \cos \alpha - V_{dU} \cos \beta}{R_{cr} + R_l + R_{ci}} \] (4.3)

for safe commutation margin, \( \gamma \) is used instead of \( \beta \)

\[ \therefore I_d = \frac{V_{dq} \cos \alpha - V_{dU} \cos \gamma}{R_{cr} + R_l - R_{ci}} \] (4.4)

\( -R_{ci} \) makes it difficult for the stable operation of the controller.

4.5 Firing Angle Control

There are basically two types of firing angle control schemes. They are

1. Individual Phase Control (IPC), and
2. Equidistant Pulse Control (EPC)

4.5.1 Individual Phase Control

The firing pulse generated for each valve is independent of each other and is determined by the zero crossing of the commutation voltage of the corresponding valve. There are two ways in which this can be accomplished. They are constant \( \alpha \) control and inverse cosine control.

**Constant \( \alpha \) control**

Since there are 6 valves, there will be six commutation voltages and six gate pulses will be generated corresponding to the zero crossing of these voltages. A block diagram for the control is shown in Figure 4.1.

![Figure 4.1: Constant \( \alpha \) control](image_url)

The output of the ZCD is a rectangular pulse of period \( \pi \). This same pulse can be used to drive the valve (after passing through a pulse amplifier). In order to control the instant at which this pulse reaches the valve, a delay circuit is used. The duration of delay is calculated from the magnitude of the control voltage \( V_c \) from the current controller. An example is shown in figure.

**Inverse Cosine Control**

4.5.2 Equidistant Pulse Control

**Advantages**

1. The scheme provides equal pulse spacing in the steady state.
2. Low non-characteristic harmonics when used with weak ac systems.

Drawbacks

1. During unbalanced ac voltages, it results in a lower dc voltage and power than IPC.

4.6 Current Control

4.7 Extinction Angle Control

Knowing commutation voltage and $\gamma_{\text{min}}$, one can find $\beta$ as shown below

$$2L \frac{di_3}{dt} = e_b - e_a = \sqrt{3}E_m \sin \omega t$$

(4.5)

$$2L \int_0^l di_3 = \sqrt{3}E_m \int_{t_1}^{t_2} \sin \omega t \, dt$$

(4.6)

$$2LI_d = -\frac{\sqrt{3}E_m}{\omega} \left[ \cos \omega t \right]_{\alpha/\omega}^{\pi-\alpha/\omega}$$

(4.7)

$$2\omega LI_d = -\sqrt{3}E_m \left[ \cos (\pi - \gamma_{\text{min}}) - \cos \alpha \right]$$

(4.8)

$$2\omega LI_d = \sqrt{3}E_m \left[ \cos \gamma_{\text{min}} + \cos \alpha \right]$$

(4.9)

$$-\cos \alpha = \cos \gamma_{\text{min}} - \frac{2\omega LI_d}{\sqrt{3}E_m} = \cos \beta$$

(4.10)

From the above expression it is clear that if $I_d$ is measured and if $\omega$, $L$, $E_m$ are known, the we can calculate the $\beta$ required.

4.8 Starting and Stopping of dc links

4.8.1 Start-up of dc link

HVDC links can be started by using either

1. long gate pulses (120°) or
2. short gate pulses (60°)

Long pulse firing

1. De-block the inverter at about $\gamma = 90^\circ$
2. De-block the rectifier at about $\alpha = 85^\circ$ to establish low direct current
3. Gradually ramp up the voltage by inverter control and the current by rectifier control.

Short pulse firing

In this case, the problem of current extinction during start up is present as the valve with forward bias is not put into conduction when the current in that transiently falls below holding current.

The starting sequence is as follows:

1. Open the bypass switch at one terminal
2. De-block that terminal and load to minimum current in the rectifier mode.
3. Open bypass switch at the second terminal and commutate current to the bypass pair.
4. Start the second terminal also in the rectifier mode
5. The inverter terminal is put into the inversion mode
6. Gradually ramp up the voltage and current

The voltage is normally raised before raising the current. This permits the insulation of the line to be checked before raising the power. The ramping of power avoids stresses on the generator shaft. The switching surges on the line are also reduced.
Chapter 5

Faults and Protection

5.1 DC Smoothing Reactors

5.1.1 Functions of smoothing reactors

The smoothing reactor performs the following functions and is essential for the converter operation in HVDC system.

1. To prevent consequent commutation failures in inverters by reducing the rate of rise of direct current during commutation in one bridge when the direct voltage of another series connected bridge collapses.

2. They reduce the incidence of commutation failure in inverters cause by dips in the ac voltage at the converter bus.

3. It smoothens the dc current ripples.

4. To reduce the harmonic voltage and current in the dc line.

5. To reduce the requirement of dc filters and ac filters.

6. To reduce the current and voltage transients, during sudden changes in dc power flow.

7. To reduce the crest of the short circuit current in the dc line, thereby reducing the stresses on the converter valve.

8. To limit the current in the valves during the converter bypass pair operation, due to the discharge of shunt capacitances of the dc line.

5.1.2 Disadvantages and Limitations

1. High inductance of smoothing reactor on dc side results in slowing down of response of current control of HVDC transmission systems.

2. Reactor has additional losses.

3. The resonant frequency is reduced and the current stabilization control becomes difficult.

4. High stored energy causes high short circuit currents on dc side between pole bus and earth.
5.1.3 Principle of smoothing effect

Smoothing reactor functions as a pure inductance. Current $I_d$ through the reactor cannot change instantaneously. Under changing current condition, the emf is induced in the reactor coil given by

$$e = L \frac{di}{dt}$$

(5.1)

This emf opposes the rate of change of current. The current change takes finite time of the order of a few micro or milli seconds. The above principle gives a smoothing effect to the dc waveform.

5.1.4 Criterion for choice of smoothing reactor

1. Inductance should be sufficient to ensure flow of dc current even when the delay angle $\alpha$ is close to $90^\circ$.
2. The inductance should smoothen the dc voltage and current waveforms to acceptable limits. The percentage of ripples should be reduced to acceptable limits.
3. During the inverter faults and dc line faults, the rate of rise of current should not exceed the limit.
4. The inductance value should remain practically constant with variations in the direct current.
5. Inductance should be low, so that the dc system current noise (due to magnetostriction) is low.

There is little information available on the choice of the optimum size of the dc smoothing reactor. One criterion used is the $S_i$ factor, defined below.

$$S_i = \frac{V_{dn}}{LI_{dn}}$$

(5.2)
Chapter 6

Harmonics and Filters

6.1 Harmonics

Harmonics other than \( np \pm 1 \) are called as non-characteristic harmonics. They are generally of low magnitudes. The causes are

1. Normally valves are not fired at equal intervals due to unbalance of 3-phase supply.
2. Even balanced circuits with jitter in electronic circuitry produce non-characteristic harmonics.
3. Controller action (especially CEA)
4. Interaction of characteristic harmonics and fundamental current in non-linear elements in power system
5. Saturation of transformers

6.2 Filters

Harmonics are induced into both ac and dc circuits. The purpose of using harmonic filters in ac circuits are

1. to reduce harmonic voltages and currents in the ac power network to acceptable levels and
2. to provide all or part of the reactive power consumed by the converter.

Whereas in dc, filters are used to reduce harmonics in the dc line.

6.2.1 Type of connection: Series or Shunt

The filters can be connected in series or shunt or both.

The series filter must carry the full current of the main circuit and must be insulated throughout for full voltage to ground.

The shunt filters can be grounded at one end and carries only the harmonic current for which it is tuned, plus a fundamental current much smaller than that of the main circuit. Hence a shunt filter is much cheaper than a series filter of equal effectiveness.

AC shunt filters have another advantage over series filters in that at fundamental frequency, the former supplies needed reactive power but the latter consumes it.

For the above said reasons, shunt filters are used exclusively on the ac side. AC filters are star connected with grounded neutral.

On the dc side, the dc reactor constitutes all or part of the dc filters. The remainder as dc shunt filters.
6.3 AC filters

AC filters are tuned for either a particular frequency (tuned filter) or a range of frequencies (damped filter). Tuned filters (high $Q^1$ factor) are sharply tuned to one or two of the lower harmonic frequencies such as fifth or seventh. Damped filters (low Q factor) offers low impedance to a wide band of high frequencies, like for example from seventeenth and higher order. Damped filters are also called as high-pass filters.

6.3.1 Types

The following are the various types of ac filters that can be used:

1. Single tuned filter
2. Double tuned filter
3. High-pass filter
   (a) Second order filter
   (b) C type filter

The single tuned filters are designed to filter out characteristic harmonics of single frequency. The double tuned filters are used to filter out two discrete frequencies, instead of using two single tuned filters. Their main advantages are:

1. its power loss at fundamental frequency is less
2. one inductor, instead of two is subjected to full impulse voltage.

The second order high-pass filters are designed to filter out the higher harmonics. The tuning of these filters is not critical. The losses at the fundamental frequency can be reduced by using a $C$ type filter where the capacitor $C_2$ in series with $L$, provides a low impedance path to the fundamental component of current.

All the filter branches appear capacitive at fundamental frequency and supply reactive power.

6.4 DC filters

As stated before, the dc reactors, although designed primarily for other functions, constitute all or part of the filters on the dc side. If the dc line is in a cable, generally no additional filtering is required on the dc side, because the cable sheath and the ground or sea water adequately shield telephone lines from noise induced by the dc harmonics. With overhead dc lines, additional filtering is usually required.

The harmonics in the dc voltage contain both characteristic and non-characteristic orders ($h = np$). These harmonics result in current harmonics in dc lines and cause noise in telephone circuits. The dc filter design procedure is similar to the ac filter design procedure except the difference that reactive power of dc filter is not significant.

The effectiveness of the dc filter is judged by one of the following criteria:

1. Maximum voltage Telephone Influence Factor (TIF$^2$) on dc high voltage bus.
2. Maximum induced noise voltage (INV) in milli volts/km in a parallel test line one kilometer away from the HVDC line.
3. Maximum permissible noise to ground in dB in telephone lines close to HVDC lines.

$^1$The sharpness of tuning is often expressed in terms of $Q$ factor.
$^2$TIF is a measure of telephone interference caused by the harmonics in the power circuit.
Chapter 7

Reactive Power Compensation

7.1 Sources of reactive power

The HVDC converts ac power to dc power. On ac side, the voltage and the current are not in phase and hence the current has got a quadrature component. Thus the converter requires reactive power compensation for the satisfactory operation. AC system supplies active power as well as reactive power. However this is not enough. Hence additional compensation is provided on ac side of converter by the following means:

1. ac filter capacitors
2. shunt capacitors
3. synchronous condensers
4. static VAr Sources (SVS)

These are shown schematically in figure.

The reactive power consumption vary mainly with the following

1. Active power $P_d$ and
2. Delay angle $\alpha$ of rectifier (and extinction angle $\gamma$ of inverter)

Reactive power demand varies from 20% to 60% of the active power flow. This requires adjustable reactive power source which can provide variable reactive power as demanded. For slow variations in the load, switched capacitors or filters can provide some control. However, this is discrete type of control and can result in voltage flicker unless the size of the unit, which is switched, is made sufficiently small. In contrast, the synchronous condensers and static VAr systems provide continuous control of the reactive power and can follow fast load changes.

7.1.1 Synchronous Condensers

Synchronous condensers are essentially synchronous motors operating at no load, with excitation control to maintain the terminal voltage. Their advantages are as follows:

1. The availability of voltage source for commutation at the inverter even if the connection to the ac system is temporarily interrupts. This also implies an increase in SCR as the fault level is increased. When the load supplied by the inverter is passive, the synchronous condenser is essential for providing voltage sources for the line commutation at the inverter.

2. Better voltage regulation during a transient due to the maintenance of flux linkages in the rotor windings. The effect of the armature reaction is counteracted during a transient by induced currents in the field and armature circuits.

Synchronous condensers also have certain disadvantages. These are:

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1. high maintenance and cost
2. possibility of instability due to the machine going out of synchronism.

The static VAr systems provide the fastest response following a disturbance. The configurations normally used are

1. fixed capacitor (FC) or thyristor controlled reactor (TCR)
2. thyristor switched capacitors (TSC)
Part II

University Exam Questions and Answers
Chapter 8

Module I

8.1 Short Answer Questions (5 marks)

1. Compare ac and dc transmission systems. (November 2009)
2. Explain the modern trends in dc transmission. (November 2009)
3. What are the disadvantages of dc transmission? (July 2010)
4. Explain the overvoltage protection of valve. (July 2010)
5. Give the advantages, disadvantages and applications of dc transmission systems. (November 2011)
6. Give a brief description about the reliability of dc transmission system. Also explain
   - Energy availability
   - Transient stability
   (November 2011)
7. What are the applications of dc transmission? (November 2010)
8. What do you mean by valve firing? (November 2010)
9. What are the types of dc links? (June 2012)
10. Explain valve design considerations. (June 2012)
11. Compare ac and dc transmission on the basis of economics of power transmission. (June 2013)
12. List five applications of HVDC transmission. (June 2013)

   Answer:
   (a) Long distance bulk power transmission
   (b) Power transmission using underground or underwater cables (for distance > 32km)
   (c) Asynchronous interconnection of AC systems operating at different frequencies or where
       independent control of systems is desired
   (d) Control and stabilisation of power flows in AC ties in an integrated power system
   (e) Multi-terminal HVDC system is used for interconnecting three or more 3-phase ac sys-
       tems.

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8.2 Long Answer Questions

1. (a) Compare the ac and dc transmission. (10 marks)
   (b) Explain the stability limits of ac power transfer. (5 marks) (July 2010)

2. (a) Explain about different valve tests. (10 marks)
   (b) Discuss about valve design considerations. (5 marks) (July 2010)

3. (a) Compare ac and dc transmission. (10 marks)
   (b) What are the types of dc links? (5 marks) (November 2010)

4. (a) Explain the modern trends in dc transmission. (10 marks)
   (b) Write short note on choice of voltage level. (5 marks) (November 2010)

5. Compare ac and dc transmission. (15 marks) (June 2011)

6. (a) Explain about the thyristor valve design considerations. (5 marks)
   (b) Discuss briefly about the different types of thyristor valve tests. (10 marks) (June 2011)

7. What are the factors which determine the relative merits of two modes (ac and dc) transmission system? Explain each. (15 marks) (November 2011)

8. Explain thyristor valve firing and the valve design considerations. (15 marks) (November 2011)

9. (a) Explain a typical HVDC converter station. (10 marks)
   (b) Differentiate between ac and dc transmission. (5 marks) (June 2012)

10. (a) Explain choice of voltage level for dc transmission. (10 marks)
    (b) Write short notes on valve firing scheme. (5 marks) (June 2012)

11. (a) Draw the schematic diagram of a typical HVDC converter station and explain the function of each unit. (10 marks)
    (b) Explain a back to back HVDC system with a neat diagram. (5 marks) (June 2013)

12. Analyse the advantages of dc transmission over ac transmission with reference to technical performance. (15 marks) (June 2013)
Chapter 9

Module II

9.1 Short Answer Questions (5 marks)

1. What do you mean by a pulse number? (July 2010, November 2009)

2. Explain the characteristics of a 12 pulse converter. (July 2010)

3. For a 12 pulse converter with \( q = 4, s = 3, r = 1 \), calculate the maximum dc power and transformer ratings (valve windings) if PIV rating of the valve is \( V \) and the rms rating is \( I \). (November 2010)

4. Define pulse number and explain pulse converter circuit. (November 2011)

5. Explain the effect of source reactance on pulse converter circuit. (November 2011)

6. Derive the expression for average maximum dc voltage and valve utilisation factor. (June 2012)

7. Write short notes on pulse number and transformer rating. (June 2012)

8. A transformer secondary line voltage to a 3 phase bridge rectifier is 345kV. Calculate the dc voltage output with overlap angle \( = 15^\circ \) and \( \alpha = 15^\circ \). (June 2013)

9. With a neat figure, write short note on converter bridge characteristics. (June 2013)

9.2 Long Answer Questions

1. Explain with neat waveforms, the working of Graetz circuit. (15 marks) (July 2010)

2. Analyze in detail, the working of converters. (15 marks) (July 2010)

3. Explain the characteristics of a 12 pulse converter. (15 marks) (November 2010)

4. Explain 2 and 3 valve conduction. (15 marks) (November 2010)

5. With suitable waveforms analyze the working of Graetz circuit with and without overlap. (15 marks) (June 2011, November 2011)

6. Explain briefly about the working of bridge converter circuit in two and three valve conduction modes. (15 marks) (June 2011)

7. Explain the rectifier and inverter characteristics of 6 pulse converter. (15 marks) (November 2011)
8. Explain Gratez circuit without overlap in detail. (15 marks) (June 2012)

9. (a) Explain 2 and 3 valve conduction (10 marks)
    (b) Write short notes on the choice of converter configuration. (5 marks) (June 2012)

10. (a) With a neat circuit diagram and waveforms, explain the working of a Graetz circuit. (12 marks)
    (b) Explain the term commutation group. (3 marks) (June 2013)

11. With a neat circuit diagram, analyse a Graetz circuit without overlap and derive the expression for the average direct voltage, PIV, valve rating and VA rating for transformer secondary. (15 marks) (June 2013)
Chapter 10

Module III

10.1 Short Answer Questions (5 marks)

1. Explain the basic principles of dc link control. (November 2009, November 2011)

2. Write short note on surge arrestors. (November 2009)

3. Explain about the current control and regulation of the voltage in the link. (July 2010)

4. What are the different types of converter faults in general? (July 2010)

   **Answer**
   According to the origin of the malfunction, converter faults can be divided into 3 broad groups.

   (a) Faults due to malfunction of valves and controllers
       i. Arc backs (or back fires) in mercury arc valves
       ii. Arc through (fire through)
       iii. Misfire
       iv. Quenching or current extinction
   (b) Commutation failures in inverters
   (c) Short circuits in a converter station

5. Explain the hierarchical control structure for a dc link. (November 2011, November 2010, June 2012)

6. Explain briefly the choice of converter configuration for any pulse number. (November 2010)

7. What are the drawbacks of IPC scheme and how can it be overcome? (November 2010)

   **Drawbacks**
   (a) Aggravation of harmonic stability problem
   (b) Higher magnitudes of non-characteristic harmonics ($h \neq np \pm 1$)
   (c) Causes perturbations in zero crossing of voltage waveforms
   (d) Non equidistant firing pulses results in non-symmetric output voltage
   (e) All the above problems are aggravated at the frequencies for which the filter impedance and the system impedance are in parallel resonance
   (f) Cannot be used in weak ac systems

   Theses problems due to harmonics instability can be overcome by

   (a) Providing additional filters to filter out the non-characteristic harmonics.
(b) Use of filters in control circuit to filter out non-characteristic harmonics in the commutation voltage. This can however be problematic due to variations in the supply frequency and will add to the control delays.
(c) Use of a firing angle control scheme which is independent of the zero crossing. This lead to the development of EPC.

8. Write short notes on misfire and commutation failure. (June 2012)
9. Explain the heirarchial control structure for a dc link with a neat diagram. (June 2013)
10. Write short note on the various causes of over-voltages in a converter station. (June 2013)

10.2 Long Answer Questions

1. a) Explain the different methods of firing angle control. (7 marks)
b) Explain about different converter faults. (8 marks) (November 2009)

2. Discuss about the overvoltages in a converter station and the protection methods against it. (15 marks) (July 2010)

3. Explain about the starting and stopping of dc link. (15 marks) (July 2010)

4. a) A monopolar HVDC link has one bridge at each terminal. The parameters of the link are:
   \( \alpha_{\text{min}} = 5^\circ, \gamma_{\text{min}} = 18^\circ, R_d = 5\Omega, R_{cr} = 10\Omega, R_{ci} = 12\Omega, V_{d0r} = 115kV, \)
   \( I_{\text{ref}} \) at the rectifier = 1kA, \( I_{\text{ref}} \) at the inverter = 900A. Calculate \( I_d, \alpha, \gamma, P_i \) and \( Q_i \) if
   (i) \( V_{d0i} = 117.5kV \)
   (ii) \( V_{d0i} = 120kV \)
   (10 marks)
b) Write short notes on surge arresters. (5 marks) (November 2010)
a) In order to determine \( I_d \), we need to know the operating mode which can be any of the three possibilities
   i. Current control at the rectifier, \( \gamma_i = \gamma_{\text{min}} \)
   ii. Current control at the inverter, \( \alpha_r = \alpha_{\text{min}} \)
   iii. \( \alpha_r = \alpha_{\text{min}}, \gamma_i = \gamma_{\text{min}} \)
   To identify the operating mode, we need to calculate
   \[
   I_d = \frac{V_{d0i} \cos \alpha_{\text{min}} - V_{d0i} \cos \gamma_{\text{min}}}{R_{cr} + R_i - R_{ci}}
   \]
   (10.1)
   If \( I_d > I_{\text{ref}} \) at the rectifier, then the operating mode is current control at the rectifier. If \( I_d < I_{\text{ref}} \) at the inverter, the operating mode is current control at the inverter. If both conditions are not met, then the operating mode is \( \alpha_r = \alpha_{\text{min}}, \gamma_i = \gamma_{\text{min}} \).
   \textbf{When} \( V_{d0i} = 117.5kV \),
   \( I_d = 937.7A \)
   The mode of operation is \( \alpha_r = \alpha_{\text{min}}, \gamma_i = \gamma_{\text{min}} \). Hence,
   \( \alpha_r = 5^\circ \)
   \( \gamma_i = 18^\circ \)
   \( V_{di} = V_{d0i} \cos \gamma_{\text{min}} - R_{ci} I_d = 100.5kV \)
   \( P_i = V_{di} I_d = 92.24MW \)
   \( \cos \phi_i = \frac{V_d}{V_{d0i}} = 0.855 \)
   \( Q_i = P_i \tan \phi_i = 57.1MV Ar \)
When $V_{d0i} = 120kV$, 

5. Explain converter faults. (15 marks) (November 2010)

6. Explain briefly about the principles of DC link control. (15 marks) (June 2011)

7. Explain briefly about different types of converter faults. (15 marks) (June 2011)

8. Explain basic converter control characteristics. What are the two requirements which necessitate the modification of the control characteristics? (15 marks) (November 2011)

9. With the help of neat diagrams and waveforms, explain Individual Phase Control (IPC) and Equidistant Pulse Control (EPC). (15 marks) (November 2011)

10. Discuss the principles of DC link control and converter control characteristics. (15 marks) (June 2012)

11. Explain firing angle control. (15 marks) (June 2012)

12. (a) Write short note on surge arrester. (5 marks)

   (b) Explain the three basic types of faults that can occur in converters. (10 marks) (June 2013)

13. With neat circuit diagram and waveforms, explain individual phase control and equidistant pulse control. (15 marks) (June 2013)
Chapter 11

Module IV

11.1 Short Answer Questions (5 marks)

1. Explain the characteristics and types of DC breakers. (November 2009)

2. Write short note on smoothing reactors. (July 2010, November 2009, June 2012)

3. Explain the sources of reactive power. (July 2010, November 2009)

4. What are the technical advantages of a STATCOM over a SVC? (November 2010)
   Answer:
   (a) Faster response (fraction of a cycle) which is independent of the network impedance.
   (b) Requires lesser space as bulky passive components (such as reactors) are eliminated.
   (c) Inherently modular and relocatable
   (d) It can be interfaced with real power sources such as battery, fuel cell or SMES (Superconducting Magnetic Energy Storage)
   (e) A STATCOM has superior performance during low voltage condition as the reactive current can be maintained constant. It is even possible to increase the reactive current in a STATCOM under transient conditions if the devices are rated for the transient overload.

5. Explain briefly about a dc line insulator. (November 2010)

6. What are the three types of ac filters? Explain each. (November 2011)
   (a) Single tuned filter
   (b) Double tuned filter
   (c) High pass filter
      i. Second order filter
      ii. C type filter

7. Give a brief description about multi terminal dc (MTDC) system and its applications. (November 2011)

8. Explain types of ac filters. (June 2012)

9. Explain any five functions of smoothing reactors. (June 2013)
   Answer:
   (a) To reduce the incidence of commutation failure in inverters caused by dips in the ac voltage at the converter bus.

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(b) To prevent consequent commutation failures in inverters by reducing the rate of rise of
direct current in the bridge when the direct voltage of another series conected bridge
 collapses.

(c) To smoothen the ripple in the direct current in order to prevent the current from
becoming discontinuous at light loads.

(d) To decrease harmonic voltages and currents in the dc line.

(e) To limit the crest current in the rectifier due to a short circuit on the dc line.

(f) To limit the current in the valves during the converter bypass pair operation, due to
the discharge of shunt capacitances of the dc line.

10. With a neat diagram, explain the general arrangement of a HVDC circuit breaker. (June
2013)

11.2 Long Answer Questions

1. (a) Discuss about the protection of DC line. (5 marks)
(b) Explain the characteristics and types of DC breakers. (10 marks)
   (November 2009)

2. Explain and discuss about the different functions of smoothing reactors. (15 marks) (July
2010)

3. Explain in detail about static VAR system. (15 marks) (July 2010)

4. Explain dc breakers. (15 marks) (November 2010)

5. (a) What are the effects of corona? (10 marks)
(b) Explain the functions of smoothing reactors. (5 marks) (November 2010)

6. Explain about the basic concepts, types and characteristics of DC breakers. (15 marks)
   (June 2011)

7. Write short note on:
   (a) Different types of multi-terminal DC system.
   (b) Control and protection of MTDC system.
   (15 marks) (June 2011)

8. Describe Thyristor Controlled Reactor (TCR) and Thyristor Switched Capacitor (TSC). (15
marks) (November 2011)

9. What are the two possible types of MIDS systems? Explain and compare these two systems.
(15 marks) (November 2011)

10. Describe the effects of Corona and DC line insulators. (15 marks) (June 2012)

11. Explain dc breakers in detail. (15 marks) (June 2012)

12. (a) Explain the various simulation tools that can be employed for the simulation of HVDC
systems. (10 marks)
(b) List any five requirements of a good simulation tool. (5 marks) (June 2013)

   Answer:
   b) The requirements of a good simulation tool are as follows:
      i. Ease of maintenance
ii. Accuracy of solution
iii. Flexibility of use
iv. Reduced cost
v. Ease of setting up
vi. Real time simulation
vii. Easy monitoring and control over the simulation process

13. (a) Briefly explain the various reactive power sources at a converter bus. (5 marks)
(b) With neat circuit diagrams, explain the working of a thyristor controlled reactor and thyristor switched capacitor. (10 marks) (June 2013)
References


Appendix A

Formulae

\[
\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad \text{(A.1)}
\]

\[
\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad \text{(A.2)}
\]
Appendix B

Derivations

B.1 Mode 2

\[ V_d = \frac{3}{\pi} \left[ \int_{u}^{u+\alpha} -\frac{3}{2} e_c \, \text{d}\omega t + \int_{u+\alpha}^{\pi/3+\alpha} e_{bc} \, \text{d}\omega t \right] \] (B.1)

\[ = \frac{3}{\pi} \frac{E_m}{2} \sin \left( \omega t - \frac{\pi}{2} \right) \, \text{d}\omega t + \sqrt{3} E_m \int_{u+\alpha}^{\pi/3+\alpha} \sin \left( \omega t + \frac{\pi}{3} \right) \, \text{d}\omega t \] (B.2)

\[ = \frac{3}{\pi} E_m \left[ -\frac{3}{2} \int_{u}^{u+\alpha} \left( \sin \omega t \cos \frac{\pi}{2} - \cos \omega t \sin \frac{\pi}{2} \right) \, \text{d}\omega t + \right. \] (B.3)

\[ \left. + \sqrt{3} \int_{u+\alpha}^{\pi/3+\alpha} \left( \sin \omega t \cos \frac{\pi}{3} + \cos \omega t \sin \frac{\pi}{3} \right) \, \text{d}\omega t \right] \]

\[ = \frac{3}{\pi} E_m \left[ \frac{3}{2} \int_{u}^{u+\alpha} \cos \omega t \, \text{d}\omega t + \frac{\sqrt{3}}{2} \int_{u+\alpha}^{\pi/3+\alpha} \sin \omega t \, \text{d}\omega t + \right. \] (B.4)

\[ \left. \frac{3}{2} \int_{u+\alpha}^{\pi/3+\alpha} \cos \omega t \, \text{d}\omega t \right] \]

\[ = \frac{3}{\pi} E_m \left[ \frac{3}{2} \int_{u}^{u+\alpha} \sin \omega t \, \text{d}\omega t + \frac{\sqrt{3}}{2} \int_{u+\alpha}^{\pi/3+\alpha} \cos \omega t \, \text{d}\omega t \right] \] (B.5)

\[ = \frac{3}{\pi} E_m \left[ \frac{3}{2} \sin \left( \frac{\pi}{3} + \alpha \right) - \frac{\sqrt{3}}{2} \cos \left( \frac{\pi}{3} + \alpha \right) \right] \] (B.6)

\[ = \frac{3}{\pi} E_m \left[ \frac{3}{2} \left( \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \sin \alpha \right) - \frac{\sqrt{3}}{2} \frac{1}{2} \cos \left( \frac{\pi}{3} + \alpha \right) \right] \] (B.7)

\[ = \frac{3}{\pi} E_m \left[ \frac{3}{2} \left( \frac{3}{4} \cos \alpha + 3 \sin \alpha + \frac{\sqrt{3}}{4} \cos (u + \alpha) + \frac{3}{4} \sin \alpha - \frac{\sqrt{3}}{4} \cos \alpha \right) \right] \] (B.8)

\[ = \frac{3}{\pi} E_m \left[ \frac{\sqrt{3}}{2} \cos \alpha + \frac{\sqrt{3}}{2} \frac{1}{2} \cos (u + \alpha) \right] \] (B.9)

\[ = \frac{3\sqrt{3}}{2\pi} E_m \left[ \cos \alpha + \cos (u + \alpha) \right] \] (B.10)

\[ = \frac{V_{d0}}{2} \left[ \cos \alpha + \cos (u + \alpha) \right] \] (B.11)
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